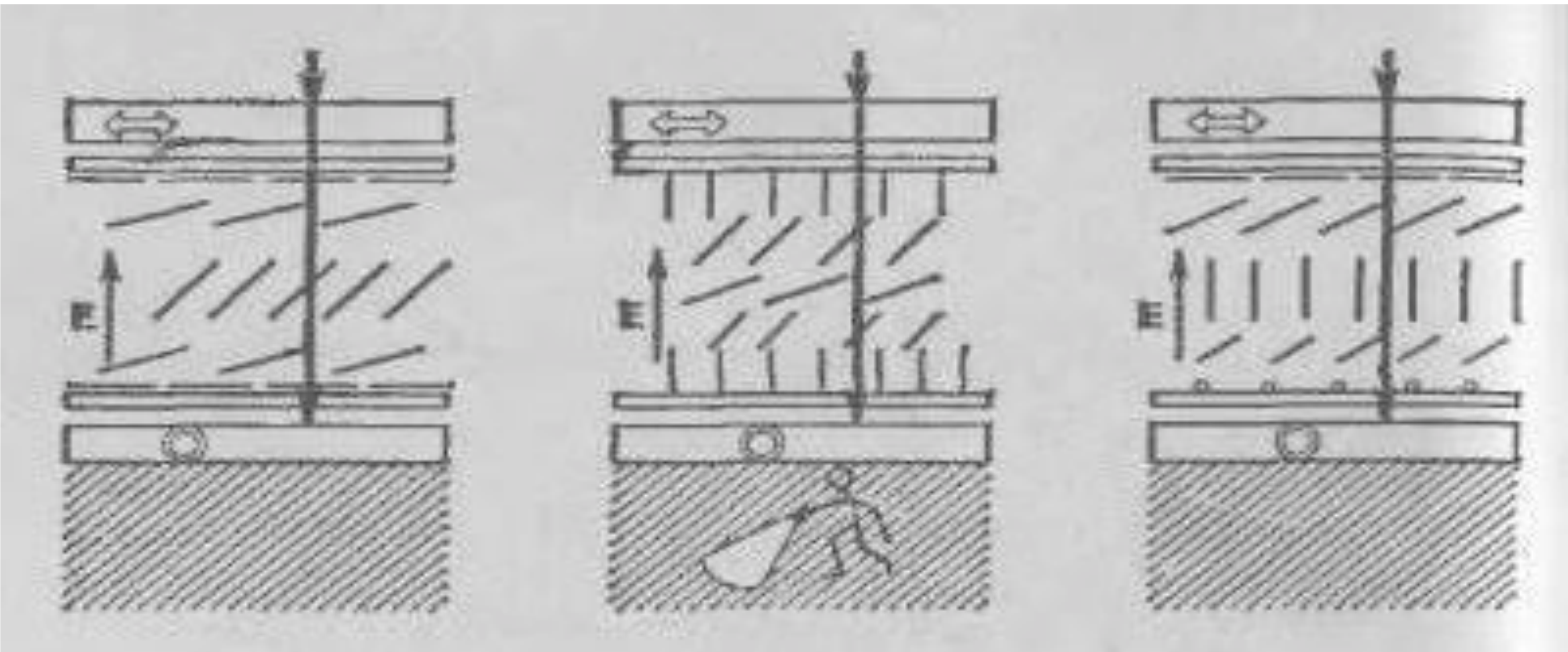
Влияние внешнего поля на пространственную структуру кирального жидкого кристалла.

Влияние поля при различной начальной ориентации молекул:



Рассматриваемая задача:

$$\boldsymbol{n_0} = (\cos\phi, \sin\phi, 0)$$

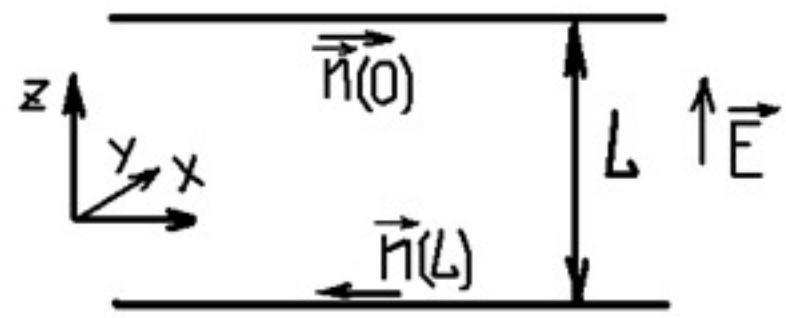
$$\phi = q_0 z + \phi_0$$

$$q_0 = \frac{2\pi}{2L}$$

$$\phi_0 = 0$$

$$\boldsymbol{n}(0) = \boldsymbol{e_x}$$

$$\boldsymbol{n}(L) = -\boldsymbol{e_x}$$



Свободная энергия Франка:

$$F = \frac{1}{2}\int dV[K_{11}\cdot(div\boldsymbol{n})^2 + K_{22}(\boldsymbol{n}\cdot rot\boldsymbol{n} + q_0)^2 + K_{33}[\boldsymbol{n}\times rot\boldsymbol{n}]^2 - \frac{\varepsilon_a}{4}(\boldsymbol{nE})^2]$$

$$\boldsymbol{E} = (0, 0, E)$$

$$\boldsymbol{n} = (\sin\theta\,\cos\phi, \sin\theta\,\sin\phi, \cos\theta)$$

$$\theta = \theta(z)$$

$$\phi = \phi(z)$$

Вид энергии после преобразования:

$$F = \frac{1}{2}\int dV \{K_{11}\sin^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 + K_{22}\left(q_0 - \sin^2\theta\cdot\frac{\partial\phi}{\partial z}\right)^2 + K_{33}\cos^2\theta\left(\left(\frac{\partial\theta}{\partial z}\right)^2 + \sin^2\theta\cdot\left(\frac{\partial\phi}{\partial z}\right)^2\right) -$$

$$-\frac{\varepsilon_a}{4\pi}\cos^2\theta\cdot E^2\}$$

$$\frac{\partial F}{\partial\theta} = \frac{d}{dz}\left(\frac{\partial F}{\partial\theta'(z)}\right)$$

$$\frac{\partial F}{\partial\phi} = \frac{d}{dz}\left(\frac{\partial F}{\partial\phi'(z)}\right)$$

Уравнения Эйлера-Лагранжа на углы приводят к выражениям:

$$\int dV\,(K_{11}\sin^2\theta \times \theta'' + K_{33}\cos^2\theta \times \theta'') =$$

$$= \int dV\,\{-\frac{K_{11}}{2}\sin 2\theta \cdot (\theta')^2 + K_{22}\sin 2\theta \cdot \phi'\left(\sin^2\theta \cdot \phi' - q_0\right) +$$

$$+ K_{33}\left(\frac{1}{2}\sin 2\theta \times (\theta')^2 + \frac{1}{2}\sin 2\theta\cos 2\theta \times (\phi')^2\right) + \frac{\varepsilon_a}{4\pi}\sin^2\theta \times E^2\}$$

$$\int dV\,(K_{22}\sin 2\theta \cdot \theta'\left(\sin^2\theta \cdot \phi' - q_0\right) + K_{22}\sin^2\theta\sin 2\theta \cdot \theta' \cdot \phi' +$$

$$+ K_{22}\sin^4\theta \times \phi'' +$$

$$+ \frac{K_{33}}{4}\sin 4\theta \times \theta' \times \phi' + \frac{K_{33}}{4}\sin^2 2\theta\phi'') = 0$$

$$\frac{d}{dz}\left(\int dV\,\{K_{22}\sin^2\theta\left(\sin^2\theta\cdot\phi' - q_0\right) + K_{33}\sin^2\theta\cos^2\theta\cdot\phi'\right) = 0$$

$$\frac{d}{dz}\left[-I_1 + I_2 - 2C\phi'\right] = 0$$

$$I_1 = \frac{1}{2}(1 - (K_{33} - K_{11})\sin^2\theta)\theta''$$

$$I_2 = \frac{1}{2}(\sin^2\theta - (K_{33} - K_{22})\sin^4\theta)\phi'' - K_{22}q_0\sin^2\theta\cdot\phi' +$$

$$+\frac{1}{2}K_{22}q_0{}^2 - \frac{\varepsilon_a}{4\pi}\cos^2\theta\cdot E^2$$

$$\phi = q_0 z + \xi(z)$$

$$\theta = \frac{\pi}{2} - \delta(z)$$

$$\int k_{11}\delta'' + 2\delta k_{22}\xi'(z)(q_0 + \xi'(z)) - \delta k_{33}(q_0 + \xi'(z))^2 + \frac{\varepsilon_a E^2}{4\pi} 2\delta = 0$$

$$k_{22}\varphi'' = 0$$