Analytical and Numerical Study of Equilibrium Characteristics of a Droplet with Charged Condensation Nucleus in the External Electric Field

A. K. Shchekin, M. S. Kshevetskii, and V. B. Varshavskii

Fock Research Institute of Physics, St. Petersburg State University,
ul. Ul'yanovskaya 1, Petrodvorets, St. Petersburg, 198504 Russia
Received October 13, 2000

Abstract—The equilibrium parameters of small dielectric droplet with charged condensation nucleus in the external uniform electric field are studied. Two typical cases are considered: (i) the droplet with charged nucleus suspended by external uniform electric field in the gravitational field and (ii) the droplet moves steadily under the action of external electric field with allowance for the resistance of surrounding vapor–gas medium. It is taken into account that the charged condensation nucleus can be displaced from the mass center of the droplet to new equilibrium position inside the droplet under the action of external electric field and response field. The scheme of the numerical solution of a nonlinear system of differential equations for the droplet equilibrium profile and electric potentials inside the droplet and in the vapor–gas medium at the arbitrary values of droplet size, strength of external field, and the charge of condensation nucleus is formulated and realized. Dependences of an equilibrium profile and the thermodynamic characteristics of a droplet such as the chemical potential of condensate and formation work on the droplet size, mass, and charge of condensation nucleus, the strength of external field and ratio of permittivities of droplet and the vapor–gas medium are plotted. Results of numerical calculations are supplemented by the analytical relations for equilibrium droplet characteristics in the first orders of the perturbation theory for a weak external field.

INTRODUCTION

The formation of droplets from supersaturated vapor on charged insoluble particles often occurs in the presence of static electric fields. These fields affect both the equilibrium profile and the thermodynamic characteristics of nucleating droplets that, in turn, influences the rates of the processes of nucleation and condensation.

Thermodynamics of nucleation on charged particles in the absence of external electric field was considered earlier in [1, 2]. The effect of external uniform electric field on the parameters of homogeneous droplets nucleating in the vapor–gas medium was described in [3–6]. Nonlinear equilibrium deformation of conducting and dielectric droplets in the external uniform field was numerically studied in [7–9]. Simulation of the formation work of small droplets in systems with various intermolecular potentials in external uniform electric field was recently conducted within the framework of Monte Carlo method [10, 11].

Main ideas of the allowance for joint effect of external electric field and the field of charged condensation nucleus in the thermodynamics of dielectric droplet have been treated in [12]. In this work, the droplet equilibrium profile, potentials of electric field in this droplet and the vapor–gas medium, as well as thermodynamic characteristics of a droplet such as the chemical potential and formation work were determined. The solution

of a problem was obtained using the perturbation theory with respect to small parameter characterizing the deviation of droplet shape from spherical and under the assumption that the displacement of condensation nucleus with respect to the mass center of the droplet can be neglected. This assumption substantially limited the domain of applicability of the results obtained in [12].

In order to generalize the approach developed in [12], we assume that the condensation nucleus is displaced from the mass center of the droplet to its new equilibrium position inside the droplet under the action of external electric field and the response field in dielectric medium. This, in turn, should affect the profile and the thermodynamic characteristics of a droplet. As a result, in the case of strong electric fields, essentially nonlinear effects are possible such as considerable nonspherical droplet deformation, the displacement of condensation nucleus from the mass center of the droplet up to its surface and even the droplet emission, as well as the loss of droplet stability related to these effects. To study similar phenomena, the perturbation theory is insufficient. Therefore, in this work we formulate the scheme of numerical solution of the system of differential equations for the equilibrium droplet profile and electric potentials inside the droplet and in the vapor–gas medium at arbitrary values of the strength of external field and the charge of condensation nucleus. Results of numerical calculations will be supplemented with the analytical relations for equilibrium character-

istics of a droplet in the first orders of perturbation theory in a weak external field.

1. FORMULATION OF A PROBLEM

Let the droplet of volume V composed of ν molecules of an incompressible dielectric liquid (the α phase) with mass density ρ^α be formed in the gaseous medium of supersaturated vapor mixed with passive gas (the β phase) on a point condensation nucleus with mass m and charge q (hereafter, superscripts α and β denote that the corresponding parameter refers to liquid or gaseous phases, respectively). Let the external electric field of strength $\mathbf{E}_\infty$, which with allowance for a rather small droplet size can assumed to be almost always uniform, acts on this system. Equilibrium shape of a droplet is determined from the pressure balance with allowance for the Maxwell tensions in any point of the droplet surface [12]

$$P_0^\alpha - P_0^\beta + P_{ex} + \frac{\varepsilon^\alpha(\mathbf{E}^\alpha)^2}{8\pi} - \frac{\varepsilon^\beta(\mathbf{E}^\beta)^2}{8\pi}$$
$$+ \frac{\varepsilon^\beta(E_N^\beta)^2}{4\pi} - \frac{\varepsilon^\alpha(E_N^\alpha)^2}{4\pi} = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right), \quad (1.1)$$

where $P_0^{\alpha,\beta}$ is the pressure in the corresponding phase in the absence of an electric field, but at the same value of chemical potential as in the presence of a field; $\varepsilon^{\alpha,\beta}$ is the permittivity of corresponding phase determined, generally speaking, as a function of chemical potential; $\mathbf{E}^{\alpha,\beta}$ and $E_N^{\alpha,\beta}$ are the vectors of electric field strength and component of this vector normal to the droplet surface, respectively; R_1 and R_2 are the principal radii of the surface curvature in a considered point; and γ is the surface tension (considering the droplet as sufficiently large, we assume that γ is a scalar quantity invariant to the presence of a field). The contribution of P_{ex} is related to the action of external forces, such as the gravity force or the resistance force of the vapor–gas medium, on the droplet. As was mentioned above, in contrast to [12], we assume that the nucleus can be displaced inside the droplet and, correspondingly, we introduce the mass m of a nucleus into consideration. As a result, under the action of the external electric field and the response field, the new equilibrium position of a nucleus in a droplet will be at a distance h from the mass center of liquid comprising droplet (at $m \ne 0$, the position of the mass center of the droplet will be different than that of the mass center of liquid).

In order to set the left-hand side of Eq. (1.1), it is necessary to solve Laplace's equation for the electric potential Φ with the boundary conditions at the droplet surface S

$$\Delta\Phi^{\alpha,\beta} = 0, \quad (1.2)$$

$$\Phi^\alpha\big|_S = \Phi^\beta\big|_S, \quad \varepsilon^\alpha(\nabla\Phi^\alpha, \mathbf{n})\big|_S = \varepsilon^\beta(\nabla\Phi^\beta, \mathbf{n})\big|_S, \quad (1.3)$$

where $\mathbf{n}$ is the unit vector of an outer normal to the droplet surface. In addition, one should take into account special boundary conditions connected with the positions of the sources of electric field.

For further analysis, we take advantage of the approach employed earlier by us [9, 13] for the numerical study of the effect of electric field on the equilibrium characteristics of dielectric droplets. For this purpose, we pass to the spherical coordinate system related to the mass center of the liquid part of a droplet (such a selection of a coordinate system is convenient for studying the deformation of the droplet profile) such that the polar axis be directed along the symmetry axis of electric field (along the vector of external field strength). Variable $x \equiv \cos\theta$ is used instead of azimuthal angle θ. In addition, we determine the dimensionless pressure drop G, charge $\mathcal{H}_q$, strength of external field $\mathcal{H}_E$, potential of electric field $\tilde{\Phi}$, pressure $\tilde{P}_{ex}$, permittivity ε, modulus of radius-vector $\tilde{r}$, droplet profile $\tilde{a}(x)$, nucleus displacement $\tilde{h}$, and mass of condensation nucleus $\tilde{m}$ using the following relations:

$$G \equiv \frac{R(P_0^\alpha - P_0^\beta)}{2\gamma} = \frac{R(\mu^\alpha - \mu_\infty)}{2\gamma v^\alpha},$$

$$\mathcal{H}_q^2 \equiv \frac{q^2(\varepsilon^\alpha - \varepsilon^\beta)}{16\pi\gamma R^3(\varepsilon^\beta)^2}, \quad \mathcal{H}_E^2 \equiv \frac{(\varepsilon^\alpha - \varepsilon^\beta)E_\infty^2 R}{16\pi\gamma},$$

$$\tilde{\Phi} \equiv \sqrt{\frac{\varepsilon^\alpha - \varepsilon^\beta}{16\pi\gamma R}}\Phi, \quad \tilde{P}_{ex} \equiv P_{ex}\frac{R}{2\gamma}, \quad \varepsilon \equiv \frac{\varepsilon^\alpha}{\varepsilon^\beta}, \quad \tilde{r} \equiv \frac{r}{R}, \quad (1.4)$$

$$\tilde{a}(x) \equiv \frac{a(x)}{R}, \quad \tilde{h} \equiv \frac{h}{R}, \quad \tilde{m} \equiv \frac{m}{\rho^\alpha V},$$

where r is the radial coordinate; μ^α and μ_∞ are the dimension chemical potentials of a molecule in a liquid phase at pressure P_0^α and at equilibrium of the liquid–vapor system with the interface, respectively; v^α is the molecular volume in a liquid; $a(x)$ is the dimension droplet profile; and R is the radius of equivalent sphere for the droplet of volume V. In this case, Eqs. (1.1)–(1.3) with allowance for Eq. (1.4) have the form

$$\Delta\tilde{\Phi}^{\alpha,\beta}(\tilde{r}, x) = 0, \quad (1.5)$$

$$\tilde{\Phi}^\alpha(\tilde{a}(x), x) - \tilde{\Phi}^\beta(\tilde{a}(x), x) = 0, \quad (1.6)$$

$$[\varepsilon(\tilde{a}^2\tilde{\Phi}_{\tilde{r}}^\alpha - (1 - x^2)\tilde{a}_x\tilde{\Phi}_x^\alpha)$$
$$- (\tilde{a}^2\tilde{\Phi}_{\tilde{r}}^\beta - (1 - x^2)\tilde{a}_x\tilde{\Phi}_x^\beta)]\big|_{\tilde{r}=\tilde{a}} = 0, \quad (1.7)$$

$$G + \tilde{P}_{ex} - \frac{1}{2\sqrt{\tilde{a}^2 + (1-x^2)\tilde{a}_x^2}}$$

$$\times \left(2 + \frac{(1-x^2)(\tilde{a}_x^2 - \tilde{a}\tilde{a}_{xx}) + x\tilde{a}\tilde{a}_x}{\tilde{a}^2 + (1-x^2)\tilde{a}_x^2} + \frac{x\tilde{a}_x}{\tilde{a}} \right)$$

$$+ \left[\frac{(\tilde{a}^2\tilde{\Phi}_r^\alpha - (1-x^2)\tilde{a}_x\tilde{\Phi}_x^\alpha)(\tilde{a}^2\tilde{\Phi}_r^\beta - (1-x^2)\tilde{a}_x\tilde{\Phi}_x^\beta)}{\tilde{a}^2(\tilde{a}^2 + (1-x^2)\tilde{a}_x^2)} \right.$$

$$\left. + \frac{(1-x^2)(\tilde{\Phi}_x^\alpha + \tilde{a}_x\tilde{\Phi}_r^\alpha)(\tilde{\Phi}_x^\beta + \tilde{a}_x\tilde{\Phi}_r^\beta)}{\tilde{a}^2 + (1-x^2)\tilde{a}_x^2} \right]\Bigg|_{\tilde{r}=\tilde{a}} = 0, \qquad (1.8)$$

where subscripts x and $\tilde{r}$ at $\tilde{a}$, $\tilde{\Phi}^\alpha$, and $\tilde{\Phi}^\beta$ denote the derivatives of these parameters with respect to x and $\tilde{r}$, respectively.

The radius of equivalent sphere R and the condition to the position of the mass center of liquid comprising droplet in the coordinate origin can be determined from two additional equations

$$\int_{-1}^{1} (\tilde{a}^3(x) - 1)dx = 0, \qquad (1.9)$$

$$\int_{-1}^{1} x\tilde{a}^4(x)dx = 0. \qquad (1.10)$$

It remains to take into account the boundary conditions. For the free droplet, derivative with respect to profile a over azimuthal angle θ satisfies conditions $a_\theta = 0$ at $\theta = 0$ and $\theta = \pi$. These conditions in variable x are fulfilled automatically. Boundary conditions to potentials related to the position of the sources of electric field are defined as

$$\tilde{\Phi}^\beta \xrightarrow[\tilde{r}\to\infty]{} -\mathcal{H}_E\tilde{r}x, \quad \tilde{\Phi}^\alpha \xrightarrow[\substack{\tilde{r}\to\tilde{h}\\x\to1}]{} \frac{\mathcal{H}_q}{\epsilon}\sum_{k\geq0}\frac{\tilde{h}^k}{\tilde{r}^{k+1}}P_k(x). \quad (1.11)$$

The first of these conditions states that the field becomes uniform at infinity, and the second condition is the expansion of the Coulomb potential of the charge of condensation nucleus in the vicinity of point $\tilde{r} = \tilde{h}$, $x = 1$ in terms of Legendre polynomials $P_k(x)$.

Further our interest is with two cases. In the first case, the droplet is quiescent and suspended by the electric filed in the presence of gravitational force. For this case, the contribution of P_{ex} to the pressure can be represented in the form

$$P_{ex} = -\rho^\alpha ga(x)x, \qquad (1.12)$$

where g is the gravitational acceleration. Note that, at given $\mathcal{H}_q$, the droplet size R and the strength of external electric field $\mathcal{H}_E$ are interrelated in this case by the con-

dition of droplet mechanical equilibrium and, hence, cannot be chosen arbitrarily. It seems convenient to choose droplet size R as independent variable, and the strength of external field $\mathcal{H}_E$ is assumed to be unknown function of $\mathcal{H}_q$ and R. In the absence of gravitational force, the g value can be considered as the droplet acceleration; in this case, the presence of P_{ex} in Eq. (1.1) can be due to the noninertia of the frame of reference related to the mass center of liquid in a droplet.

In the second case, the droplet moves steadily with the velocity **u** under the action of external electric field and the resistance force of the vapor–gas medium (in this case, the gravitational force is neglected). Assuming that the droplet sizes are small compared with the free path of the molecules of the vapor–gas medium (flowing around the droplet occurs in a free-molecular regime), we can represent the contribution of P_{ex} to the pressure as

$$P_{ex} = -\frac{2\rho_g v_t}{\sqrt{\pi}}(\mathbf{u}, \mathbf{n}), \qquad (1.13)$$

where ρ_g is the gas mass density and v_t is the gas thermal velocity. Note that, in this case, at given $\mathcal{H}_q$, the droplet size R and the strength of external electric field $\mathcal{H}_E$ can be chosen arbitrarily. The velocity u of steady-state motion of a droplet is assumed to be the unknown function of $\mathcal{H}_q$, $\mathcal{H}_E$, and R.

Let us determine dimensionless gravitational acceleration $\tilde{g}$ and velocity $\tilde{u}$ of a droplet as

$$\tilde{g} \equiv \frac{g\rho^\alpha R^2}{2\gamma\mathcal{H}_q}, \quad \tilde{u} = \frac{\rho_g v_t R u}{\gamma\sqrt{\pi}\mathcal{H}_q}, \qquad (1.14)$$

then, using Eqs. (1.4) and (1.12)–(1.14), we present $\tilde{P}_{ex}$ in the first and second cases, respectively, as

$$\tilde{P}_{ex} = -\tilde{g}\mathcal{H}_q\tilde{a}x \qquad (1.15)$$

and

$$\tilde{P}_{ex} = -\tilde{u}\mathcal{H}_q\frac{\tilde{a}x - \tilde{a}_x(1-x^2)}{\sqrt{\tilde{a}^2 + (1-x^2)\tilde{a}_x^2}}. \qquad (1.16)$$

Finally, we arrive at the system of equations whose form is close to that obtained in [12]. The unknown values of this system are G, $\tilde{a}(x)$, $\tilde{\Phi}^\alpha(\tilde{r}, x)$, $\tilde{\Phi}^\beta(\tilde{r}, x)$, and $\tilde{h}$, as well as $\mathcal{H}_E$ (the first case) and $\tilde{u}$ (the second case). All other values ϵ, $\tilde{m}$, $\mathcal{H}_q$, and correspondingly, $\tilde{g}$ (the first case) and $\mathcal{H}_E$ (the second case) are the parameters of a problem and can be set arbitrarily. It is seen that Eqs. (1.5)–(1.7) and boundary conditions (1.11) are used for the formal definition of $\tilde{\Phi}^\alpha(\tilde{r}, x)$ and $\tilde{\Phi}^\beta(\tilde{r}, x)$; Eqs. (1.8)–(1.10), (1.15) or (1.16), for finding four remaining unknown values $\tilde{a}(x)$, G, $\tilde{h}$, and

$\mathcal{H}_E$ or $\tilde{u}$. It was assumed [12] that $\tilde{h} = 0$; then the system of Eqs. (1.5)–(1.11) appeared to be closed. It is not so in our case.

To close system (1.5)–(1.11), we assume that the resultant force acting on a condensation nucleus in its equilibrium position in a droplet is equal to zero. In general, we have (for the case of a droplet moving steadily, we assume formally that $g \equiv 0$)

$$mg - F_{\text{el}} = 0, \qquad (1.17)$$

where $F_{\text{el}} \equiv -q(\nabla \Phi_R^\alpha)\big|_{\substack{r = h \\ x = 1}}$ is the force acting on the

charge from the side of electric field and Φ_R^α is the potential of response field equals potential Φ^α minus the Coulomb field of charged nucleus. In dimensionless variables (1.4) and (1.14), this equation acquires the form

$$\tilde{m}\tilde{g} + \frac{6}{\varepsilon - 1}(\nabla \tilde{\Phi}_R^\alpha)\big|_{\substack{\tilde{r} = \tilde{h} \\ x = 1}} = 0. \qquad (1.18)$$

Note that, in the case of steadily moving droplet at $\tilde{g} = 0$, the mass m of condensation nucleus is not present explicitly in Eqs. (1.5)–(1.10) and (1.18). This means that the results obtained are applicable to the condensation nuclei of arbitrary mass.

2. NUMERICAL SCHEME

To solve systems of Eqs. (1.5)–(1.10) and (1.18), let us expand the droplet profile $\tilde{a}(x)$ and potentials of electric field $\tilde{\Phi}(\tilde{r}, x)$ into series in terms of Legendre polynomials $P_k(x)$ with allowance for boundary conditions (1.11)

$$\tilde{a}(x) = \sum_{k \geq 0} d_k P_k(x), \quad \tilde{\Phi}_R^\alpha(\tilde{r}, x) = \sum_{k \geq 0} c_k \tilde{r}^k P_k(x),$$

$$\tilde{\Phi}^\alpha(\tilde{r}, x) = \frac{\mathcal{H}_q}{\varepsilon} \sum_{k \geq 0} \frac{\tilde{h}^k}{\tilde{r}^{k+1}} P_k(x) + \tilde{\Phi}_R^\alpha(\tilde{r}, x), \qquad (2.1)$$

$$\tilde{\Phi}^\beta(\tilde{r}, x) = -\mathcal{H}_E \tilde{r} x + \sum_{k \geq 0} \frac{b_k}{\tilde{r}^{k+1}} P_k(x),$$

where coefficients $\{d_k\}$, $\{c_k\}$, and $\{b_k\}$ ($k = 0, 1, 2, \ldots$) have to be found. When using expansions (2.1), Eqs. (1.5) are transformed into identities. Substituting Eq. (2.1) with allowance for Eqs. (1.15) or (1.16) into Eqs. (1.6)–(1.8), multiplying consequently the left-hand side of each equation by Legendre polynomials $P_n(x)$, $n = 0, 1, 2, \ldots$, and integrating with respect to x in the range $x \in [-1, 1]$, we reduce them to algebraic equations for expansion coefficients (2.1). Equations (1.9), (1.10), and (1.18) are reduced to similar equations after substituting of Eq. (2.1). As a result, we arrive at the infinite

system of nonlinear equations with respect to the sets of unknown coefficients $\{b_k\}$, $\{c_k\}$, and $\{d_k\}$, $k = 0, 1, 2, \ldots$, as well as G, $\tilde{h}$, and t values (by t is meant $\mathcal{H}_E$ or $\tilde{u}$, depending on the considered problem)

$$\int_{-1}^{1} w_1(x; G, t, \tilde{h}, \{b_k\}, \{c_k\}, \{d_k\}) P_n(x) dx = 0,$$

$$\int_{-1}^{1} w_2(x; G, t, \tilde{h}, \{b_k\}, \{c_k\}, \{d_k\}) P_n(x) dx = 0,$$

$$\int_{-1}^{1} w_3(x; G, t, \tilde{h}, \{b_k\}, \{c_k\}, \{d_k\}) P_n(x) dx = 0,$$

$$w_4(G, t, \tilde{h}; \{b_k\}, \{c_k\}, \{d_k\}) = 0, \qquad (2.2)$$

$$w_5(G, t, \tilde{h}; \{b_k\}, \{c_k\}, \{d_k\}) = 0,$$

$$w_6(G, t, \tilde{h}; \{b_k\}, \{c_k\}, \{d_k\}) = 0,$$

where w_1, w_2, w_3, w_4, w_5, and w_6 are the left-hand sides of Eqs. (1.6)–(1.10) and (1.18) with allowance for Eqs. (1.15) or (1.16) after substituting the values from Eq. (2.1) into these equations, and index n runs values $0, 1, 2, \ldots$.

This system is equivalent to the initial system of Eqs. (1.6)–(1.10) and (1.18), but in contrast to the first system, it is more suitable for the numerical simulation, because it can be easily "cut." In Eq. (2.1), we confine ourselves only to the first N terms of expansion, whereas the remaining coefficients b_k, c_k, and d_k are assumed to be equal to zero at $k \geq N$. Hence, in system (2.2) we remain only those equations where $n < N$.

As a result, we obtain the final system of $3(N + 1)$ equations with respect to coefficients b_k, c_k, and d_k, $0 \leq k < N$, as well as with respect to G, $\tilde{h}$, and t values. The system obtained is solved minimizing the residual by the Newton method [14]. The accuracy of solution can be estimated varying N value. According to estimates, the required accuracy is achieved at $N = 30$.

Recalling that, in the case of suspended droplet, the droplet size R is chosen as an independent variable, in accordance with Eqs. (1.4) and (1.14), we obtain the requirement for the recalculation of parameters $\tilde{m}$, $\mathcal{H}_q$, and $\tilde{g}$ at each given R.

Knowing the droplet profile and electric potentials inside and outside the droplet, we can calculate the work of droplet formation. Let us denote by $\tilde{W}$ the formation work of droplet with charged nucleus in the external field normalized to $4\pi\gamma R^2$ (i.e., the formation work of the surface of homogeneous droplet of the same size in the absence of field). Omitting the droplet-size-independent contribution due to the work of transferring charged nucleus to the liquid phase (contribu-

tion of the work of nucleus wetting) and the standard contribution related to the chemical potential of a vapor, we have

$$\tilde{W} = \tilde{W}_s + \tilde{W}_{el} + \tilde{W}_g, \qquad (2.3)$$

where $\tilde{W}_{el}$ is the work of electric field, $\tilde{W}_s$ is the formation work of droplet surface, and $\tilde{W}_g$ is the work required to displace the condensation nucleus to the height $\tilde{h}$ in the gravitational field with respect to the mass center of liquid in a droplet. Dimensionless work $\tilde{W}_g$ is written as

$$\tilde{W}_g = \frac{mgh}{4\pi\gamma R^2} = \frac{2}{3}\mathcal{H}_q \tilde{m}\tilde{g}\tilde{h}. \qquad (2.4)$$

In the case of a steadily moving droplet, $g = 0$ and, correspondingly, $\tilde{W}_g \equiv 0$. Using Eqs. (7.7) and (7.4) from [12], let us represent $\tilde{W}_s$ and $\tilde{W}_{el}$ in variables of Eq. (1.4) as

$$\tilde{W}_s = \frac{1}{2}\int_{-1}^{1}\tilde{a}\sqrt{\tilde{a}^2 + (1 - x^2)\tilde{a}_x^2}\,dx, \qquad (2.5)$$

$$\tilde{W}_{el} = \frac{2}{\varepsilon - 1}\left(\mathcal{H}_q\tilde{\Phi}_R^\alpha\Big|_{\substack{\tilde{r} = \tilde{h} \\ x = 1}} - \mathcal{H}_E b_1\right), \qquad (2.6)$$

where b_1 is the coefficient in expansion of $\tilde{\Phi}^\beta$ in Eq. (2.1). After substituting expansions (2.1) into (2.5) and (2.6), we arrive at the relations that express $\tilde{W}_s$ and $\tilde{W}_{el}$ via the $\tilde{h}$ value and coefficients b_1, $\{c_k\}$, and $\{d_k\}$; $k = 0, 1, 2, \dots$.

3. ANALYTICAL RELATIONS

If the strength of external field is not high and the droplet deformation is relatively small, the solution to system (1.5)–(1.10) [with allowance for Eqs. (1.15) or (1.16) and (1.18)] can be sought analytically in the form of series in powers of small parameter. As a small parameter, it is formally convenient to use the dimensionless strength of external electric field $\mathcal{H}_E$ (for the case of suspended droplet, it implies that by setting $\mathcal{H}_E$, we obtain $\tilde{g} = \tilde{g}(\mathcal{H}_E)$; and recalling further that, according to the first of relations (1.14), $\tilde{g} = \tilde{g}(R)$, we find corresponding droplet radius R); then, values $\tilde{h}$, G, $\tilde{a}(x)$, $\tilde{\Phi}^\alpha(\tilde{r}, x)$, $\tilde{\Phi}^\beta(\tilde{r}, x)$, and y (where y is $\tilde{g}$ or $\tilde{u}$, depending on the problem) can be presented as

$$G = \sum_{i \geq 0} G^{(i)}\mathcal{H}_E^i, \quad y = \sum_{i \geq 0} y^{(i)}\mathcal{H}_E^i, \quad \tilde{h} = \sum_{i \geq 0} \tilde{h}^{(i)}\mathcal{H}_E^i,$$

$$\tilde{a}(x) = \sum_{i \geq 0}\mathcal{H}_E^i\sum_{k=0}^{i} d_k^{(i)}P_k(x),$$

$$\tilde{\Phi}_R^\alpha(\tilde{r}, x) = \sum_{i \geq 0}\mathcal{H}_E^i\sum_{k=0}^{i} c_k^{(i)}\tilde{r}^k P_k(x), \qquad (3.1)$$

$$\tilde{\Phi}^\alpha(\tilde{r}, x) = \frac{\mathcal{H}_q}{\varepsilon}\sum_{k \geq 0}\frac{\tilde{h}^k}{\tilde{r}^{k+1}}P_k(x) + \tilde{\Phi}_R^\alpha(\tilde{r}, x),$$

$$\tilde{\Phi}^\beta(\tilde{r}, x) = -\mathcal{H}_E\tilde{r}x + \sum_{i \geq 0}\mathcal{H}_E^i\sum_{k=0}^{i}\frac{b_k^{(i)}}{\tilde{r}^{k+1}}P_k(x),$$

where the desired coefficients are $\{G^{(i)}\}$, $\{y^{(i)}\}$, $\{\tilde{h}^{(i)}\}$, $\{d_k^{(i)}\}$, $\{c_k^{(i)}\}$, and $\{b_k^{(i)}\}$ ($i = 0, 1, \dots; k = 0, \dots, i$).

Using expansions (3.1) in system (1.6)–(1.10) [with allowance for (1.15)] and in Eq. (1.18), equating coefficients at the identical powers of parameter $\mathcal{H}_E$, for the case of suspended droplet we obtain in the first orders with respect to $\mathcal{H}_E$

$$\tilde{g} = \frac{6\mathcal{H}_E}{(\tilde{m} + 1)(\varepsilon - 1)}, \qquad (3.2)$$

$$\tilde{h} = \frac{\varepsilon[3 - \tilde{m}(\varepsilon - 1)]}{2\mathcal{H}_q(\varepsilon - 1)(\tilde{m} + 1)}\mathcal{H}_E + O(\mathcal{H}_E^3), \qquad (3.3)$$

$$G = 1 - \frac{\mathcal{H}_q^2}{\varepsilon} - \frac{3\mathcal{H}_E^2}{\varepsilon + 2} - \frac{6\mathcal{H}_q^2\tilde{h}^2}{\varepsilon(\varepsilon + 2)} + O(\mathcal{H}_E^4), \qquad (3.4)$$

$$\tilde{a}(x) = 1 + d_2^{(2)}\mathcal{H}_E^2 P_2(x) + O(\mathcal{H}_E^3), \qquad (3.5)$$

$$\tilde{\Phi}^\beta(\tilde{r}, x) = -\mathcal{H}_E\tilde{r}x + \frac{\mathcal{H}_q}{\tilde{r}}$$
$$- \frac{\mathcal{H}_E[\tilde{m}(\varepsilon - 1) - 2\varepsilon - 1]}{2(\tilde{m} + 1)(\varepsilon - 1)\tilde{r}^2}P_1(x) \qquad (3.6)$$
$$+ \frac{\mathcal{H}_q[2d_2^{(2)}(\varepsilon - 1)\mathcal{H}_E^2 + 5\tilde{h}^2]}{(3 + 2\varepsilon)\tilde{r}^3}P_2(x) + O(\mathcal{H}_E^3),$$

$$\tilde{\Phi}^\alpha(\tilde{r}, x) = \frac{\mathcal{H}_q}{\varepsilon}\sum_{k=0}^{2}\frac{\tilde{h}^k}{\tilde{r}^{k+1}}P_k(x) + \mathcal{H}_q\frac{\varepsilon - 1}{\varepsilon} - \frac{\tilde{m}\tilde{r}x}{\tilde{m} + 1}\mathcal{H}_E$$
$$(3.7)$$
$$- \frac{3\mathcal{H}_q(\varepsilon - 1)(d_2^{(2)}\mathcal{H}_E^2 - \tilde{h}^2)}{(3 + 2\varepsilon)\varepsilon}\tilde{r}^2 P_2(x) + O(\mathcal{H}_E^3),$$

where

$$d_2^{(2)} = \frac{1}{\varepsilon(2\varepsilon + 3) - 2(\varepsilon - 6)\mathcal{H}_q^2}\left[15\frac{\mathcal{H}_q^2\tilde{h}^2}{\mathcal{H}_E^2}\right.$$

$$+ \frac{(2\varepsilon + 3)[\tilde{m}(\varepsilon - 1) + 4\varepsilon - 1]\mathcal{H}_q\tilde{h}}{2(\varepsilon - 1)(\tilde{m} + 1)}\frac{}{\mathcal{H}_E} \quad (3.8)$$

$$- \frac{\tilde{m}\varepsilon(2\varepsilon + 3)[\tilde{m}(\varepsilon - 1) - 2\varepsilon - 1]}{2(\tilde{m} + 1)^2(\varepsilon - 1)} \Bigg].$$

Substituting obtained results into Eqs. (2.3)–(2.6) and considering relations

$$b_1 = b_1^{(1)}\mathcal{H}_E + O(\mathcal{H}_E^3)$$

$$= \frac{3\mathcal{H}_q\tilde{h} + (\varepsilon - 1)\mathcal{H}_E}{\varepsilon + 2} + O(\mathcal{H}_E^3),$$

$$\tilde{\Phi}_R^\alpha\Big|_{\substack{\tilde{r} = \tilde{h} \\ x = 1}} = \mathcal{H}_q\frac{\varepsilon - 1}{\varepsilon} - \frac{3\mathcal{H}_E\tilde{h}}{\varepsilon + 2} \quad (3.9)$$

$$+ \frac{2(\varepsilon - 1)\mathcal{H}_q\tilde{h}^2}{\varepsilon(\varepsilon + 2)} + O(\mathcal{H}_E^4),$$

followed from (2.1), (3.1), (3.3), (3.6), and (3.7), we find

$$\tilde{W} = 1 + 2\frac{\mathcal{H}_q^2}{\varepsilon} - \frac{2\mathcal{H}_E^2}{\varepsilon + 2} - \frac{12\mathcal{H}_q\mathcal{H}_E\tilde{h}}{(\varepsilon - 1)(\varepsilon + 2)}$$

$$+ \frac{4\mathcal{H}_q^2\tilde{h}^2}{\varepsilon(\varepsilon + 2)} + \frac{2}{3}\mathcal{H}_q\tilde{m}\tilde{g}\tilde{h} + O(\mathcal{H}_E^4). \quad (3.10)$$

In the case of steadily moving droplet and using expansions (3.1) in system (1.6)–(1.10) [with allowance for Eq. (1.16)] and in Eq. (1.18), we obtain, respectively

$$\tilde{u} = \frac{6\mathcal{H}_E}{\varepsilon - 1} + O(\mathcal{H}_E^3), \quad (3.11)$$

$$\tilde{h} = \frac{3\varepsilon\mathcal{H}_E}{2(\varepsilon - 1)\mathcal{H}_q} + O(\mathcal{H}_E^3), \quad (3.12)$$

$$G = 1 - \frac{\mathcal{H}_q^2}{\varepsilon} - \frac{3\mathcal{H}_E^2}{\varepsilon + 2} - \frac{6\mathcal{H}_q^2\tilde{h}^2}{\varepsilon(\varepsilon + 2)} + O(\mathcal{H}_E^4), \quad (3.13)$$

$$\tilde{a}(x) = 1 - \frac{3\varepsilon(8\varepsilon^2 + 55\varepsilon - 3)\mathcal{H}_E^2}{4(\varepsilon - 1)^2[2\mathcal{H}_q^2(\varepsilon - 6) - \varepsilon(2\varepsilon + 3)]}P_2(x)$$

$$+ O(\mathcal{H}_E^3), \quad (3.14)$$

$$\tilde{\Phi}^\alpha(\tilde{r}, x) = \frac{\mathcal{H}_q}{\varepsilon}\sum_{k=0}^{2}\frac{\tilde{h}^k}{\tilde{r}^{k+1}}P_k(x) + \mathcal{H}_q\frac{\varepsilon - 1}{\varepsilon}$$

$$+ \frac{3[(7\varepsilon - 1)\mathcal{H}_q^2 - 3\varepsilon^2]\mathcal{H}_E\tilde{h}}{2\varepsilon[2\mathcal{H}_q^2(\varepsilon - 6) - \varepsilon(2\varepsilon + 3)]}\tilde{r}^2 P_2(x) + O(\mathcal{H}_E^3), \quad (3.15)$$

$$\tilde{\Phi}^\beta(\tilde{r}, x) = -\mathcal{H}_E\tilde{r}x + \frac{\mathcal{H}_q}{\tilde{r}} + \frac{(2\varepsilon + 1)x\mathcal{H}_E}{2(\varepsilon - 1)\tilde{r}^2} \quad (3.16)$$

$$- \frac{2\mathcal{H}_q^2(4\varepsilon^2 + 10\varepsilon + 1) + 15\varepsilon^2}{3\varepsilon[2\mathcal{H}_q^2(\varepsilon - 6) - \varepsilon(2\varepsilon + 3)]\tilde{r}^3}\mathcal{H}_q\tilde{h}^2 P_2(x) + O(\mathcal{H}_E^3).$$

Substituting the results obtained into Eqs. (2.3)–(2.6) and considering relations

$$b_1 = b_1^{(1)}\mathcal{H}_E + O(\mathcal{H}_E^3)$$

$$= \frac{3\mathcal{H}_q\tilde{h} + (\varepsilon - 1)\mathcal{H}_E}{\varepsilon + 2} + O(\mathcal{H}_E^3),$$

$$\tilde{\Phi}_R^\alpha\Big|_{\substack{\tilde{r} = \tilde{h} \\ x = 1}} = \mathcal{H}_q\frac{\varepsilon - 1}{\varepsilon} - \frac{3\mathcal{H}_E\tilde{h}}{\varepsilon + 2} \quad (3.17)$$

$$+ \frac{2(\varepsilon - 1)\mathcal{H}_q\tilde{h}^2}{\varepsilon(\varepsilon + 2)} + O(\mathcal{H}_E^4),$$

followed from (2.1), (3.1), (3.12), (3.16), and (3.15), we have

$$\tilde{W} = 1 + 2\frac{\mathcal{H}_q^2}{\varepsilon} - \frac{2\mathcal{H}_E^2}{\varepsilon + 2} - \frac{12\mathcal{H}_q\mathcal{H}_E\tilde{h}}{(\varepsilon - 1)(\varepsilon + 2)}$$

$$+ \frac{4\mathcal{H}_q^2\tilde{h}^2}{\varepsilon(\varepsilon + 2)} + O(\mathcal{H}_E^4). \quad (3.18)$$

Note that the appearance of Eqs. (3.4) and (3.13) as well as Eqs. (3.9) and (3.17) are resembling, although $\tilde{h}$ is determined in these equations by different Eqs. (3.3) and (3.12). In this sense, expression (3.10) also agrees with Eq. (3.18), if one takes into account that $\tilde{g} \equiv 0$ for steadily moving droplet. Analyzing Eqs. (3.3) and (3.12) for $\tilde{h}$, it is seen that, at $\tilde{m} \equiv 0$, they become identical; hence, the results for pressure drop G, formation work $\tilde{W}$, and displacement of condensation nucleus $\tilde{h}$ are completely identical in both cases under consideration. Note that expressions for $\tilde{h}$ set by Eqs. (3.3) and (3.12) can be derived with the use of Eqs. (3.10) and (3.18), respectively, by finding the extremum of the formation work from the value of $\tilde{h}$.

4. DISCUSSION OF THE RESULTS FOR A SUSPENDED DROPLET

Let us first discuss formula (3.2). Writing this formula in dimension variables, we obtain the condition of droplet equilibrium in the form

$$(M + m)g = qE_\infty. \quad (4.1)$$

Recalling that $\tilde{m} = m/M$, $M = \rho^\alpha(4/3)\pi R^3$, we obtain the relation between acceleration g, electric field E_∞, and

droplet size R. In this case, $\mathcal{H}_E = \mathcal{H}_E(R)$, $\mathcal{H}_q = \mathcal{H}_q(R)$, and $\tilde{m} = \tilde{m}(R)$.

In order to compare these results with those obtained in [12], it is reasonable to turn back to dimension variables. At nucleus displacement h, chemical potential μ^α, and total formation work $W = 4\pi\gamma R^2 \tilde{W} - (\mu^\beta - \mu_\infty)\nu + W_n$, where μ^β is the chemical potential of vapor molecules, and W_n is the work of condensation nucleus wetting, from Eqs. (1.4), (3.3), (3.4), and (3.10), we find

$$h = \frac{3\varepsilon^\beta - \dfrac{m}{M}(\varepsilon^\alpha - \varepsilon^\beta)}{(\varepsilon^\alpha - \varepsilon^\beta)\left(1 + \dfrac{m}{M}\right)} \frac{R^3 \varepsilon^\alpha E_\infty}{2q}, \tag{4.2}$$

$$\mu^\alpha = \mu_\infty + \frac{2\gamma v^\alpha}{R} - \frac{q^2 v^\alpha}{8\pi R^4}\left(\frac{1}{\varepsilon^\beta} - \frac{1}{\varepsilon^\alpha}\right)$$
$$- \frac{3E_\infty^2 v^\alpha}{16\pi} \frac{\left[(\varepsilon^\alpha - \varepsilon^\beta)\dfrac{m}{M} - \varepsilon^\beta\right]^2 + 2\varepsilon^\alpha\varepsilon^\beta}{(\varepsilon^\alpha - \varepsilon^\beta)\left(1 + \dfrac{m}{M}\right)^2}, \tag{4.3}$$

Note that formula (4.6) can be also derived directly from Eq. (4.4), using the common thermodynamic relationship (8.10) from [12]: $\partial W/\partial \nu = \mu_\nu - \mu^\beta$.

It was suggested [12] that the condensation nucleus is situated in the mass center of the droplet; hence, relations (4.3) and (4.4) should be transformed into analogous results set by Eqs. (6.6) and (7.10) from [12] only at $h = 0$. As can be seen from Eq. (4.2), $h = 0$ at $m/M = 3\varepsilon^\beta/(\varepsilon^\alpha - \varepsilon^\beta)$. If this relationship between the masses of a nucleus and a droplet is taken into account in Eqs. (4.3) and (4.4), these equations are actually transformed (with allowance for corresponding designations) into Eqs. (6.6) and (7.10) from [12] in the principal order with respect to external field. Note that, at $m = 0$, $h > 0$, and one cannot pass to equations reported in [12].

As follows from Eq. (4.2), the value of displacement h can be either positive or negative, depending on the sign

$$W = -(\mu^\beta - \mu_\infty)\nu + 4\pi\gamma R^2 + \frac{q^2}{2R}\left(\frac{1}{\varepsilon^\beta} - \frac{1}{\varepsilon^\alpha}\right)$$
$$- \frac{E_\infty^2 R^3}{4} \frac{\left[(\varepsilon^\alpha - \varepsilon^\beta)\dfrac{m}{M} - \varepsilon^\beta\right]^2 + 2\varepsilon^\alpha\varepsilon^\beta}{(\varepsilon^\alpha - \varepsilon^\beta)\left(1 + \dfrac{m}{M}\right)^2} + W_n. \tag{4.4}$$

When the size of condensation nucleus R_n is much larger than droplet size R, we have $W_n = -(1/\varepsilon^\beta - 1/\varepsilon^\alpha)(q^2/2R_n)$.

Chemical potential μ^α differs from the chemical potential μ_ν of the molecule in a droplet, because the explicit dependence of the mass M of a liquid in a droplet is present in relation (4.3). When one molecule is added to the droplet composed of ν molecules, g varies in view of (4.1) together with mass M [as was already commented with respect to Eq. (1.12), this value can be taken as the droplet acceleration]. Therefore, upon the addition of a molecule to a droplet, one should account for the variation in the state of remaining molecules in a droplet with varying g

$$\mu_\nu = \mu^\alpha + \nu \frac{\partial \mu^\alpha}{\partial g}\frac{\partial g}{\partial \nu} = \mu^\alpha + M\frac{\partial \mu^\alpha}{\partial M}. \tag{4.5}$$

Respectively, from Eqs. (4.3) and (4.5), we find

$$\mu_\nu = \mu_\infty + \frac{2\gamma v^\alpha}{R} - \frac{q^2 v^\alpha}{8\pi R^4}\left(\frac{1}{\varepsilon^\beta} - \frac{1}{\varepsilon^\alpha}\right)$$
$$- \frac{3E_\infty^2 v^\alpha}{16\pi} \frac{(\varepsilon^\alpha - \varepsilon^\beta)\dfrac{m^2}{M^2}\left((\varepsilon^\alpha - \varepsilon^\beta)\dfrac{m}{M} - \varepsilon^\alpha - 3\varepsilon^\beta\right) + \varepsilon^\beta(2\varepsilon^\alpha + \varepsilon^\beta)\left(3\dfrac{m}{M} + 1\right)}{(\varepsilon^\alpha - \varepsilon^\beta)\left(1 + \dfrac{m}{M}\right)^3}. \tag{4.6}$$

of difference $3\varepsilon^\beta - \dfrac{m}{M}(\varepsilon^\alpha - \varepsilon^\beta)$. This means that the condensation nucleus can be displaced with respect to the mass center of liquid both in the field direction (if the charge is positive) and in the opposite direction. Because $M \sim R^3$, for small droplets, the condensation nucleus is displaced against the field direction, whereas, for larger droplets it is always displaced in the field direction. This is clearly illustrated by Fig. 1. In Fig. 1, as in subsequent Figs. 2–4, calculations refer to the droplet of water; moreover, each droplet size corresponds to different values of the strength of external electric field. It was taken in the calculations that $q = 1.6 \times 10^{-19}$ C, $\varepsilon^\alpha = 81$, $\varepsilon^\beta = 1$, $\rho^\alpha = 10^3$ kg m^{-3}, $\gamma = 73$ mN m^{-1}, and $g = 9.8$ m s^{-2}.

Note that, for displacement $\tilde{h}$, one cannot be confined only to the linear approximation with respect to external field, because the condensation nucleus can

approach rather closely the droplet boundary (see Fig. 2); nevertheless, even in this case, the droplet profile does not virtually differ from the sphere. Curves *1* and *2* in Fig. 2 describe the displacement of condensation nucleus with respect to droplet center obtained while solving system (2.2) with usual and increased accuracy, respectively. Curve *3* refers to the analytical solution obtained from Eq. (4.2) with allowance for Eq. (4.1), and curve *4* describes the position of droplet boundary. As is seen from this figure, analytical results for describing the displacement of condensation nucleus can be applied only to small ($R \lesssim 0.3$ μm) droplets, whereas numerical calculations are applicable at $R \lesssim 1$ μm. In view of equilibrium condition (4.1), this implies that the analytical description is applicable at $E_\infty \lesssim 7$ kV m^{-1}, and the numerical results, at $E_\infty \lesssim 250$ kV m^{-1}. Also note that curve *2* virtually fits curve *4*; the intersection of curves *1* and *4*, which could denote possible emission of condensation nucleus from the droplet, is indeed a consequence of the insufficient accuracy of calculations near the droplet boundary.

Figure 3 presents the numerically calculated dependence of pressure drop G on the droplet size R at $m = 0$. This dependence is analytically described with the use of Eq. (3.4); moreover, the plot of this dependence in Fig. 3 agrees exactly with the numerical solution. Analyzing dependence (3.4) and taking into account that $\mathcal{H}_q$ is large only for small droplets and $\mathcal{H}_E$ is small over the entire range of sizes, we obtain that terms that are dependent on the value of external electric field can be neglected compared with unity, while term $-\mathcal{H}_q^2/\varepsilon$ is large only for small droplets and decreases rapidly with an increase in R [this fact is responsible for the initial rise and consequent achievement of the constant of $G(R)$ function]. Hence, it is seen that the domain on applicability of Eq. (3.4) is noticeably wider than that of Eq. (4.2).

Considering expression for the work $\tilde{W}$ of droplet formation, we arrive at the following conclusions. As was already mentioned, $\mathcal{H}_q$ is large only for small droplets, and $\mathcal{H}_E$ is small over the entire range of sizes. Therefore, only constant contribution related to the formation of the surface of spherical droplet and the contribution proportional to the squared charge of condensation nucleus are remained in Eq. (3.10). With respect to the latter contribution, it can be stated that it rapidly decreases with an increase in R. The plot of corresponding dependence obtained numerically at $m = 0$ is shown in Fig. 4. Note that the complete agreement between analytical and numerical results is observed in this case as well; hence, the applicability of Eq. (3.10) is much larger than that of Eq. (4.2).

According to Eq. (3.5), the droplet profile $a(x)$ is a symmetric function of x. However, solving this problem analytically up to the third order of magnitude with respect to $\mathcal{H}_E$, it can be easily demonstrated that the droplet profile is no longer symmetric. Figures 5a and
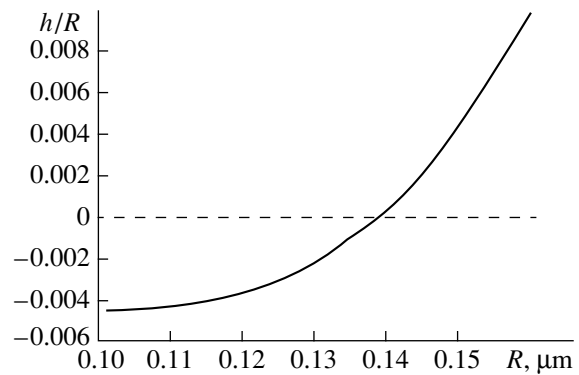

Fig. 1. Relative displacement of condensation nucleus in the droplet of water as a function of its size. Mass of condensation nucleus $m = 4.2 \times 10^{-19}$ kg. Curves obtained analytically by Eq. (4.2) and calculated numerically [from the solution of system (2.2)] coincide with each other.
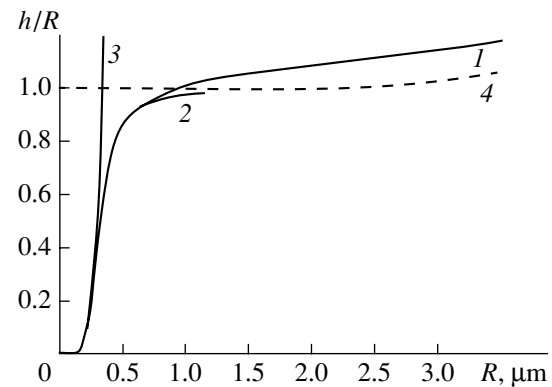

Fig. 2. Relative displacement of condensation nucleus in the droplet of water as a function of its size. Mass of condensation nucleus $m = 0$.
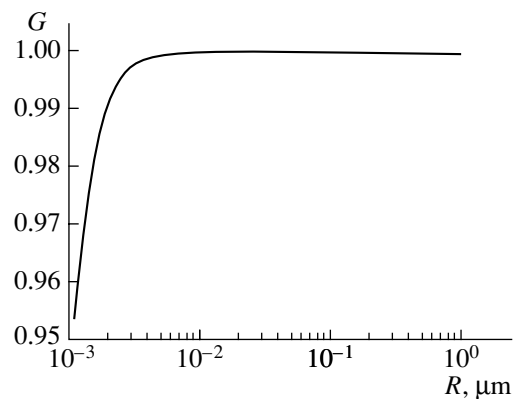

Fig. 3. Dependence of pressure drop G on the droplet size for suspended droplet. Curves obtained analytically by Eq. (3.4) and calculated numerically [from the solution of system (2.2)] coincide with each other.

5b show characteristic droplet profiles in the case of very strong electric fields obtained by the numerical solution of system (2.2). It is seen that the droplet profile is asymmetric in the field direction. The sphere of
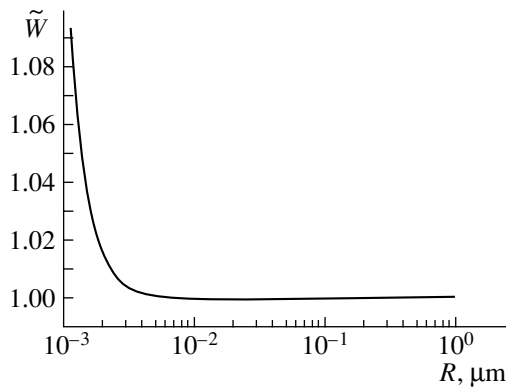
Fig. 4. Dependence of the formation work $\tilde{W}$ of water droplet on its size. Curves obtained analytically by Eq. (3.10) and calculated numerically [from the solution of system (2.2)] coincide with each other.
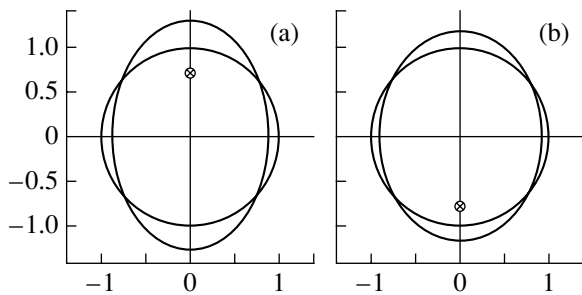

Fig. 5. Profiles of a suspended droplet in strong electric field ($\mathcal{H}_E = 1.1$, $\mathcal{H}_q = 1.0$, $\varepsilon = 25$). (a) The case of small mass of condensation nucleus ($\tilde{m} = 0.1$, $\tilde{h} = 0.67$); (b) the case of large mass of condensation nucleus ($\tilde{m} = 0.3$, $\tilde{h} = -0.59$). Small circled cross denotes the equilibrium positions of nucleus in a droplet.
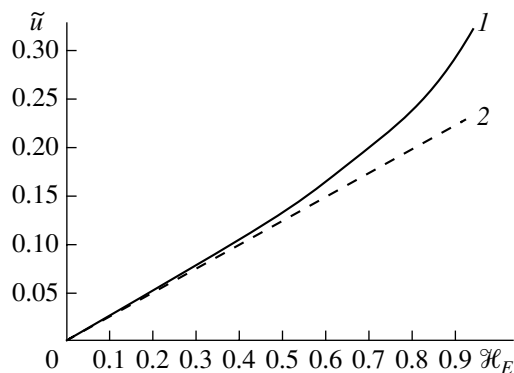
Fig. 6. Velocity $\tilde{u}$ of droplet motion as a function of external electric field $\mathcal{H}_E$ at $\mathcal{H}_q = 1$ and $\varepsilon = 25$.

the same volume is shown in these figures for comparison.

5. DISCUSSION OF THE RESULTS FOR STEADILY MOVING DROPLET

When analyzing the case of steadily moving droplet, note once again that now the droplet size R is not

related to the strength of external field E_∞; in addition, the solution is independent of the mass m of condensation nucleus. Passing to the dimension variables for droplet velocity u, nucleus displacement h, chemical potential μ^α, and total work of droplet formation $W = 4\pi\gamma R^2 \tilde{W} - (\mu^\beta - \mu_\infty)\nu + W_n$, from Eqs. (1.4), (3.11)–(3.13), and (3.18), we find

$$u = \frac{3qE_\infty}{8\sqrt{\pi}\rho_r v_\tau R^2}, \tag{5.1}$$

$$h = \frac{3\varepsilon^\alpha\varepsilon^\beta R^3 E_\infty}{2(\varepsilon^\alpha - \varepsilon^\beta)q}, \tag{5.2}$$

$$\mu^\alpha = \mu_v = \mu_\infty + \frac{2\gamma v^\alpha}{R}$$

$$- \frac{q^2 v^\alpha}{8\pi R^4}\left(\frac{1}{\varepsilon^\beta} - \frac{1}{\varepsilon^\alpha}\right) - \frac{3E_\infty^2 v^\alpha}{16\pi}\frac{\varepsilon^\beta(2\varepsilon^\alpha + \varepsilon^\beta)}{\varepsilon^\alpha - \varepsilon^\beta}, \tag{5.3}$$

$$W = -(\mu^\beta - \mu_\infty)\nu + 4\pi\gamma R^2$$

$$+ \frac{q^2}{2R}\left(\frac{1}{\varepsilon^\beta} - \frac{1}{\varepsilon^\alpha}\right) - \frac{E_\infty^2 R^3}{4}\frac{\varepsilon^\beta(2\varepsilon^\alpha + \varepsilon^\beta)}{\varepsilon^\alpha - \varepsilon^\beta} + W_n. \tag{5.4}$$

It was accounted for in Eq. (5.3) that, in contrast to the problem with suspended droplet, in the problem under consideration, chemical potential μ^α coincides with that of molecule in a droplet.

Figure 6 presents the dependence of the velocity $\tilde{u}$ of droplet motion on the value of dimensionless external electric field $\mathcal{H}_E$ obtained using numerical solution of system (2.2) (curve *1*) and formula (3.11) (curve *2*). Slight deviation of curve *1* from curve *2* at large values of $\mathcal{H}_E$ is apparently explained by the improvement of droplet streamlining.

The dependence of the relative displacement $\tilde{h}$ of condensation nucleus on the strength of external electric field $\mathcal{H}_E$ is shown in Fig. 7 (curve *1*). The initial part of the plot is adequately described by curve *2* obtained using formula (3.12). As is seen from Fig. 7, the analytical solution is applicable only for weak electric fields. Note that, in contrast to the previous case, the condensation nucleus can be displaced with respect to the mass center of liquid only in one direction (along the field, if the charge of nucleus is positive). Also note that, in the case of strong fields, the condensation nucleus can closely approach the droplet boundary.

Figures 8 and 9 show the dependences of pressure drop G and the work $\tilde{W}$ of droplet formation, respectively. The results of numerical calculation are presented in these figures by curves *1*; and analytical results, by curves *2*. As is seen, corresponding analytical and numerical curves virtually coincide with each other. According to Eq. (3.13), the pressure drop G is
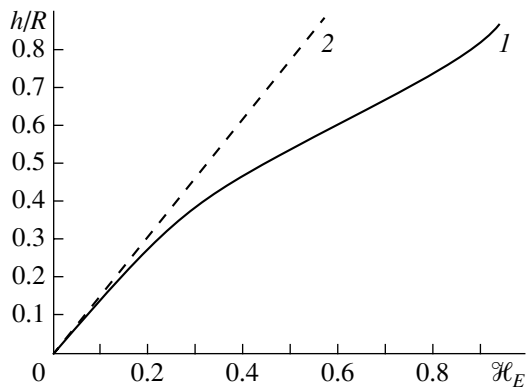
Fig. 7. Relative displacement of condensation nucleus $\tilde{h}$ as a function of external electric field $\mathcal{H}_E$ at $\mathcal{H}_q = 1$ and $\varepsilon = 25$. Curve *1* was plotted by the results of numerical solution of system (2.2); curve *2*, by formula (3.12).
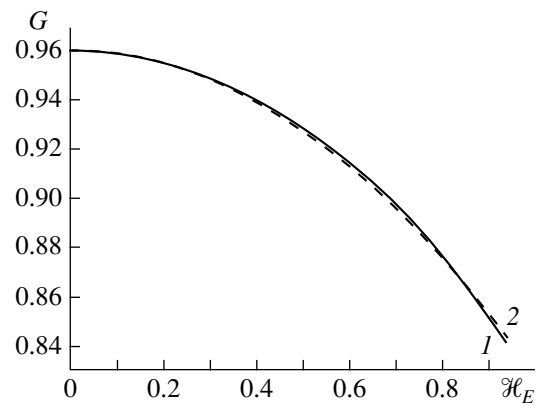

Fig. 8. Dependence of pressure drop G on the strength of external electric field $\mathcal{H}_E$ at $\mathcal{H}_q = 1$ and $\varepsilon = 25$ for steadily moving droplet. Curve *1* was plotted by the results of numerical solution of system (2.2); curve *2*, by formula (3.13).
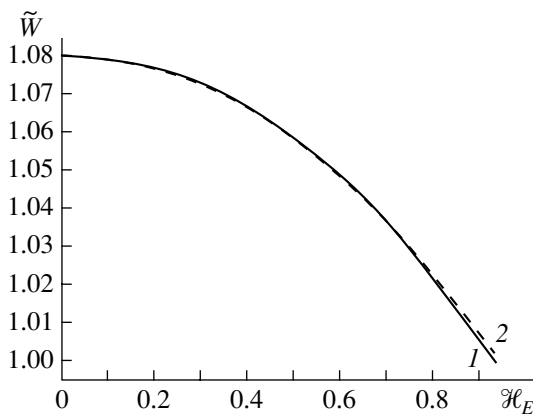

Fig. 9. Dependence of the work $\tilde{W}$ of droplet formation on the strength of external electric field $\mathcal{H}_E$ at $\mathcal{H}_q = 1$ and $\varepsilon = 25$. Curve *1* was plotted by the results of numerical solution of system (2.2); curve *2*, by formula (3.18).
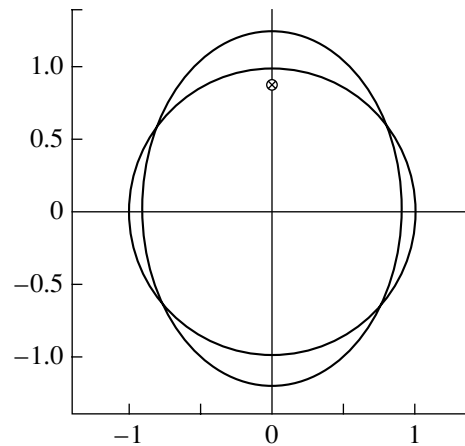

Fig. 10. Profile of steadily moving droplet in strong electric field $\mathcal{H}_E = 0.93$, $\mathcal{H}_q = 1.0$, $\varepsilon = 25$, and $\tilde{h} = 0.87$.

the even function of variables $\mathcal{H}_E$ and $\mathcal{H}_q$, which has the maximum at $\mathcal{H}_q = 0$, $\mathcal{H}_E = 0$. Hence, when either $\mathcal{H}_q$ or $\mathcal{H}_E$ increases, the chemical potential drops (Fig. 8). As is seen from Eq. (3.18), the work $\tilde{W}$ of droplet formation is also the even function of variables $\mathcal{H}_E$ and $\mathcal{H}_q$; an increase in the charge $\mathcal{H}_q$ of condensation nucleus leads to a rise in $\tilde{W}$ (however, the total work W of droplet formation decreases that is related to the contribution of the work of wetting W_n), but an increase in external field $\mathcal{H}_E$ leads to a decrease in $\tilde{W}$ (Fig. 9).

Figure 10 shows the characteristic droplet profile at high strength of external electric field ($\mathcal{H}_E = 0.93$, $\mathcal{H}_q = 1.0$, $\varepsilon = 25$, and $\tilde{h} = 0.87$). It is seen that the droplet profile is asymmetric in the field direction (sphere of the same volume is shown in this figure for compari-

son) that is manifested only in the third-order expansion of droplet profile $a(x)$ in terms of $\mathcal{H}_E$.

REFERENCES

1. Kuni, F.M., Shchekin, A.K., and Rusanov, A.I., *Kolloidn. Zh.,* 1983, vol. 45, no. 4, p. 682; no. 5, p. 901.

2. Shchekin, A.K., Rusanov, A.I., and Kuni, F.M., *Kolloidn. Zh.*, 1984, vol. 46, no. 3, p. 535.

3. Miksis, M.J., *Phys. Fluids*, 1981, vol. 24, no. 11, p. 1967.

4. Cheng, K.J., *Phys. Lett. A*, 1984, vol. 106, no. 9, p. 403.

5. Cheng, K.J. and Chaddock, J.B., *Phys. Lett. A*, 1984, vol. 106, p. 51.

6. Warshavsky, V.B. and Shchekin, A.K., *Colloids Surf. A,* 1999, vol. 148, no. 3, p. 283.

7. Basaran, O.A. and Scriven, L.E., *Phys. Fluids A,* 1989, vol. 1, no. 5, p. 799.

8. Wohlhuter, F.K. and Basaran, O.A., *J. Magn. Magn. Mater.*, 1993, vol. 122, p. 259.

9. Shchekin, A.K., Kshevetskiy, M.S., and Warshavsky, V.B., *Aerosol Sci. Technol.*, 2002, vol. 36, no. 3, p. 318.

10. Oh, K.J., Gao, G.T., and Zeng, X.C., *J. Chem. Phys.*, 1998, vol. 108, p. 4683.

11. Gao, G.T., Oh, K.J., and Zeng, X.C., *J. Chem. Phys.*, 1999, vol. 110, p. 2533.

12. Varshavskii, V.B. and Shchekin, A.K., *Kolloidn. Zh.*, 1999, vol. 61, no. 5, p. 624.

13. Shchekin, A.K., Warshavsky, V.B., and Kshevetskiy, M.S., *Nucleation and Atmospheric Aerosols 2000: 1Int. Conf.,* Rolla, Missouri, 2000, Hale, B.N. and Kulmala, M., Eds., Melville: American Inst. of Physics, 2000, p. 299.

14. Dennis, J., Jr. and Schnabel, R., *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Englewood Cliffs: Prentice-Hall, 1983. Translated under the title *Chislennye metody bezuslovnoi optimizatsii i resheniya nelineinykh uravnenii*, Moscow: Mir, 1988.