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KINETICS OF THE ESTABLISHMENT OF STEADY HOMOGENEOUS CONDENSATION OF A SUPERSATURATED VAPOR

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The kinetics of the establishment of steady homogeneous condensation is investigated on the basis of an equation for the size spectrum of nucleating centers valid at both high and low supersaturations. In each unit of volume of the system it is assumed that there is a source of condensing matter of constant power and a sink for the nucleating centers of the new phase that reach a definite size. Conditions are formulated under which the steady process will be a stable final state of the system. It is shown that the microscopic correction in the expression for the work of formation of a nucleating center associated with the dependence of the surface tension on the curvature of the surface of the nucleating center plays a decisive role in determining the values of the parameters of the system that admit the establishment of a steady process. The characteristic times of establishment of the steady process are determined.

Introduction

The subject of the present paper is the establishment in a nonequilibrium system of nucleating centers of a steady size distribution of the centers. The study of the conditions of establishment of universal distributions is of great interest in the kinetics of phase transitions of the first kind. In [1], Lifshitz and Slezov found the nucleating center distribution that arises during the final stage in the evolution of an ensemble of nucleating centers, and they called this stage the coalescence stage. Under certain conditions, which are formulated in the present paper, a steady process of growth of nucleating centers can be a stable final state of the system and, thus, an alternative to the coalescence process.

To be specific, the physical system we consider will be drops of liquid formed from a supersaturated vapor in the atmosphere of a passive gas. In each unit of volume of the system we assume there is a source of the vapor of constant power and a sink for drops that reach a definite size. It is assumed that the mean free path of the vapor molecules is much greater than the diameters of the drops formed in it. Since the practical realization of the homogeneous condensation mechanism requires a high supersaturation of the vapor in the system, we base our investigation on a microscopically derived equation for the size distribution function of the drops that is valid at both low and high supersaturation. The temperature of the gas—vapor—drop system during the condensation process is assumed to be constant.

1. Equation for the Size Distribution Function of the Drops

We consider a supersaturated single-component vapor. We define the degree of supersaturation ζ of the vapor by $\zeta=(n_1-n_{1,\infty})/n_{1,\infty}$, where n_1 is the density of the vapor (the number of molecules in unit volume), and $n_{1,\infty}$ is the density of the saturated vapor above the flat surface of the liquid phase. As independent variable to describe the size spectrum of the drops we take the number ν of molecules in a drop.

Let $n_\nu(t)$ be the number of drops consisting of ν molecules in unit volume of the system at the time t. An important auxiliary entity in the theory is the equilibrium size distribution n_ν^0 of the drops. It arises as a boundary condition to the equation for $n_\nu(t)$ in the region of small sizes (see [2]) and plays a decisive role in finding the flux of molecules evaporating from the surface of a drop. The explicit expression for $n_\nu^{(0)}$, as is clear from general thermodynamic arguments, is

$$n_\nu^{(0)} = n_1 \exp(-F_\nu),\tag{1.1}$$

where F_ν is the work of formation of a drop consisting of ν molecules expressed in units of kT, where k is Boltzmann's constant, and T is the absolute temperature. For the above relationship between the mean free path of the vapor molcules and the drop sizes, we define F_ν in accordance with

$$F_\nu = a\nu^{2/3} - a\gamma\nu^{1/3} - b\nu,\tag{1.2}$$

in which

$$a = \frac{4\pi\sigma}{kT}\left(\frac{3\nu}{4\pi}\right)^{2/3},\tag{1.3}$$

$$b = \ln(\zeta+1),\tag{1.4}$$

γ is a positive coefficient of the order of unity, σ is the specific surface tension, and v is the volume per molecule in the liquid. The second term on the right-hand side of (1.2) is the leading microscopic correction to the work of formation of the drop surface, and it is due to the dependence of the surface tension on the curvature of the surface.

In accordance with (1.1) and (1.2), the function $n_\nu^{(0)}$ for b > 0 has a minimum at the point $\nu = \nu_c$, where ν_c is the critical number of molecules in a drop and it has the value

$$\nu_c = \nu_{c,0}[1 - {}^3/_2 \gamma \nu_{c,0}^{-1/3} + O(\nu_{c,0}^{-2/3})],$$

where

$$\nu_{c,0} = (2a/3b)^3.\tag{1.5}$$

Because the correction terms in the expression for ν_c are relatively small, we shall henceforth ignore the difference between ν_c and $\nu_{c,0}$ unless we need to calculate the exponential function of the work of formation of a critical drop, i.e., $\exp(F_{\nu_c}) = \exp({}^1/_3 a\nu_{c,0}^{2/3} - a\gamma\nu_{c,0}^{1/3})$. We introduce the half-width $\Delta\nu_c$ of the minimum, defining it as the magnitude of the deviation $|\nu - \nu_c|$ from the point ν_c at which the value of the function $n_\nu^{(0)}$ is e times greater than the minimal value. We find directly from (1.1) and (1.2) that

$$\Delta\nu_c = 3a^{-1/2}\nu_c^{2/3}.\tag{1.6}$$

It can be shown that under the conditions

$$\Delta\nu_c \gg 1,\tag{1.7}$$

$$\Delta\nu_c/\nu_c \ll 1,\tag{1.8}$$

whose fulfillment is assumed in what follows, the following evolution equation holds for the size distribution function of the drops in the region $\nu \geq \nu_1$, where $\nu_1 \sim \nu_c - \Delta\nu_c$:

$$\frac{\partial n_\nu(t)}{\partial t} = -\frac{\partial}{\partial\nu}\left\{A_\nu n_\nu(t) - B_\nu \frac{\partial n_\nu(t)}{\partial\nu}\right\}\tag{1.9}$$

where

$$A_\nu = 3t_0^{-1}\nu^{2/3}[\zeta+1 - \exp({}^2/_3 a\nu^{-1/3})],\tag{1.10}$$

$$B_\nu = {}^3/_2 t_0^{-1}\nu^{2/3}[\zeta+1 + \exp({}^2/_3 a\nu^{-1/3})],\tag{1.11}$$

$$t_0^{-1} = \frac{\alpha u}{12}n_{1,\infty}(4\pi)^{1/3}(3\vartheta)^{2/3},\tag{1.12}$$

u is the mean thermal velocity, and α is the condensation coefficient. Note that the characteristic time t_0 defined by Eq. (1.12) is equal in order of magnitude to the mean free time of the molecules of the saturated

vapor.

Equation (1.9) is a local conservation law for the number of drops in the system with flux $j_\nu(t)$ of drops along the size axis given by

$$j_\nu(t) = A_\nu n_\nu(t) - B_\nu \partial n_\nu(t)/\partial \nu. \tag{1.13}$$

This equation was introduced in [3], though without specification of its limits of applicability [3]. For $|\nu - \nu_c| < \Delta\nu_c$ or $b \ll 1$, it goes over into the well-known Zel'dovich–Frenkel equation [2].

We formulate the boundary conditions for Eq. (1.9). It can be asserted that for $\nu \ll \nu_c$ an equilibrium size distribution of the drops corresponding to the instantaneous supersaturation is rapidly established in the system. As can be seen from analysis of the steady distribution, a significant deviation from the equilibrium spectrum appears only for $\nu > \nu_c - \Delta\nu_c$, when the removal of drops by the flux j_ν to the region of large sizes becomes important. Therefore, as boundary condition at small sizes we take

$$n_\nu(t)\,|_{\nu=\nu_1} = n_{\nu_1}^{(0)}. \tag{1.14}$$

Assuming further that in the considered systems the drops, having reached a certain characteristic size with $\nu = \nu_2$, where

$$\nu_2 \gg \nu_c, \tag{1.15}$$

leave the volume occupied by the supersaturated vapor, we write the boundary condition in the region of large sizes in the form

$$n_\nu(t)\,|_{\nu=\nu_2} = 0. \tag{1.16}$$

An important special case of Eq. (1.9) is the steady distribution function $n_\nu^{(s)}$ corresponding to steady flux $j^{(s)}$ of drops along the size axis. For $n_\nu^{(s)}$, in accordance with (1.13), (1.14), and (1.16),

$$n_\nu^{(s)} = j^{(s)} \int_\nu^{\nu_2} \frac{d\nu'}{B_{\nu'}} \exp\left\{ -\int_\nu^{\nu'} d\nu'' \frac{A_{\nu''}}{B_{\nu''}} \right\}. \tag{1.17}$$

In accordance with (1.14) and (1.17), the steady flux $j^{(s)}$ for given supersaturation is

$$j^{(s)} = n_{\nu_1}^{(0)} \left[\int_{\nu_1}^{\nu_2} \frac{d\nu'}{B_{\nu'}} \exp\left\{ -\int_{\nu_1}^{\nu'} d\nu'' \frac{A_{\nu''}}{B_{\nu''}} \right\} \right]^{-1}. \tag{1.18}$$

The integral in (1.18) can be calculated by the method of steepest descent by virtue of the condition (1.8). After simple transformations that take into account Eqs. (1.10), (1.11), and (1.2)–(1.6), we obtain

$$j^{(s)} = (a/\pi)^{1/2} n_{1,\infty} t_0^{-1} (\zeta+1)^2 \exp\,(-F_{\nu c}). \tag{1.19}$$

If we know the number J_0 of vapor molecules supplied in unit time to unit volume of the system, then the flux $j^{(s)}$ can be determined from the stationarity condition

$$J_0 = j^{(s)} \nu_2. \tag{1.20}$$

In this case, Eqs. (1.19) and (1.20) are to be regarded as equations for finding the steady value of the degree of supersaturation $\zeta^{(s)}$ and the corresponding critical number $\nu_c^{(s)}$ of molecules in a drop. Using (1.2)–(1.8), we obtain in the zeroth approximation in the small parameter ν_c^{-1}

$$\nu_c^{(s)} \simeq \left[\frac{3}{a} \ln[(a/\pi)^{1/2} n_{1,\infty} \nu_2/t_0 J_0] \right]^{1/2}. \tag{1.21}$$

In the complete range of variation of ν, proceeding from (1.17), we can obtain simpler approximate expressions for $n_\nu^{(s)}$. Thus, in the interval

$$\nu_s \leqslant \nu < \nu_2, \tag{1.22}$$

where ν_s is a value of ν satisfying the condition $\nu_s \gg \nu_c$, we find

$$n_\nu^{(s)} \simeq j^{(s)} t_0 / 3\nu^{2/3} \zeta. \tag{1.23}$$

The asymptotic behavior (1.23) is suitable in practice until $\nu \simeq \nu_2$ since the boundary condition (1.16) has influence only in a small neighborhood $(\nu_2 - \nu \lesssim (\zeta+2)/2\zeta)$ of the point ν_2.

Investigating Eq. (1.9), we readily discover the main features in the establishment of the steady distribution (1.17) under conditions when the supersaturation of the vapor is kept constant. It can be shown

that a steady distribution $n_\nu^{(s)}$ and, therefore, a steady flux $j^{(s)}$ are formed successively, beginning with small values of ν, and in the interval $1 < \nu \le \nu_s$ this process is completed during a time t_s of order

$$t_s \sim \nu_s \Delta\nu_c / B_{\nu_c}. \tag{1.24}$$

Further, the boundary $\nu(t)$ determining the region $\nu < \nu(t)$ in which a steady distribution has been established by the time t moves with constant velocity [4] $d\nu(t)/dt = 2j^{(s)}/n_{\nu(t)}^{(s)} - A_{\nu(t)}$. When $\nu \ge \nu_s$, this velocity goes over in accordance with (1.10) and (1.23) to the asymptotic behavior

$$d\nu(t)/dt \simeq 3\nu^{\frac{2}{3}}\zeta/t_0. \tag{1.25}$$

Using (1.25), we estimate the time t_{ν_2} during which the steady distribution is established in the interval (1.22):

$$t_{\nu_2} = t_0 \nu_2^{\frac{1}{3}}/\zeta. \tag{1.26}$$

An important result, which makes it possible to simplify the following calculations, is also that the diffusion term in Eq. (1.9) can be ignored when $t \ge t_s$ in the region $\nu > \nu_s$.

This process of establishment of a steady distribution is realized, as we have already said, if a steady supersaturation of the vapor is maintained in the volume of the system. We use what we have done above and investigate the more general case when the establishment of a steady distribution occurs simultaneously with the establishment of steady supersaturation.

2. Equation for the Critical Radius

We augment the evolution equation for the size distribution function of the drops by the matter conservation law in the form

$$J_0 = n_{1,\infty} \frac{d\zeta(t)}{dt} + \frac{dN(t)}{dt} + J_2(t), \tag{2.1}$$

where $N(t)$ is the total number of molecules in the drops at the time t, defined in accordance with the formula

$$N(t) \simeq \int_0^{\nu_2} d\nu\, \nu n_\nu(t), \tag{2.2}$$

and $J_2(t)$ is the number of molecules which leave unit volume of the system in unit time in drops consisting of ν_2 molecules. We assume that the steady value $\nu_c^{(s)}$ corresponding to the given J_0 and ν_2 satisfies the condition (1.15). We begin our consideration by taking a certain instant of time at which the instantaneous value of $\nu_c(t)$ also satisfies the condition (1.15) and the bulk of the liquid phase is in drops with $\nu \gg \nu_c(t)$. Therefore, when studying the region of sizes that is most important for the integral (2.2) we can ignore the diffusion term in the evolution equation (1.9) and simplify the coefficient in the flux term.

In the simplified equation we go over to the dimensionless radius ρ defined by

$$\rho = \nu^{\frac{1}{3}} \tag{2.3}$$

and to the new unknown function $g(\rho, t)$, which is related to $n_\nu(t)$ by

$$g(\rho, t) = 3\nu^{\frac{2}{3}}n_\nu(t). \tag{2.4}$$

From (1.9)-(1.11), (2.3), and (2.4), we obtain

$$\frac{\partial g(\rho,t)}{\partial t} = -\zeta(t)t_0^{-1}\frac{\partial g(\rho,t)}{\partial \rho}. \tag{2.5}$$

We denote by ρ_c, $\rho_c^{(s)}$, $\rho(t)$, ρ_s, ρ_2 the values of the radius ρ corresponding to the values ν_c, $\nu_c^{(s)}$, $\nu(t)$, ν_s, ν_2 of the variable ν. Equation (2.5) holds in the interval

$$\rho_s \le \rho < \rho_2, \tag{2.6}$$

which corresponds to the interval (1.22) of variation of the variable ν. If in the considered system the relative change in the critical radius ρ_c during the time t_s is small, which we assume for the time being, then the value of the unknown function $g(\rho, t)$ at the point ρ_s can always be assumed equal to its steady value for the current supersaturation. Thus, we arrive at the boundary condition

$$g(\rho, t)\,|_{\rho=\rho_s} = g^{(s)}(t), \tag{2.7}$$

where the time argument t of the function $g^{(s)}(t)$ indicates that the steady distribution is taken at the current supersaturation. Solving Eq. (2.5) with the boundary condition (2.7), we find

$$g(\rho, t) = g^{(s)}(\tilde{t}), \qquad (2.8)$$

where $\tilde{t} \equiv \tilde{t}(\rho, t)$ is the retardation time, which depends on ρ and t and is determined from the equation

$$\int_{\tilde{t}}^{t} dt' t_0^{-1}\zeta(t') = \rho. \qquad (2.9)$$

For the function $g^{(s)}(t)$ we obtain from (1.23) with allowance for (1.19) and (2.4)

$$g^{(s)}(t) = (a/\pi)^{1/2} n_{1,\infty} \zeta_-^{-1}(t) [\zeta(t)+1]^2 \exp(-F_{\rho c(t)}). \qquad (2.10)$$

We differentiate (2.2) with respect to the time and transform the derivative of its right-hand side using the evolution equation (1.9). Then, using (1.13), we obtain

$$\frac{dN(t)}{dt} = -\rho_2{}^3 j_{\rho_2}(t) + 3\int_0^{\rho_2} d\rho\, \rho^2 j_\rho(t). \qquad (2.11)$$

The first term on the right-hand side of (2.11) is the flux of matter from unit volume of the system in unit time, i.e., $\rho_2{}^3 j_{\rho_2}(t) = J_2(t)$. We consider the integral on the right-hand side of (2.11). Since its value is basically determined by the behavior of the integrand in the interval (2.6), it follows from (2.5) and (2.8) that we can write

$$3\int_0^{\rho_2} d\rho\, \rho^2 j_\rho(t) \simeq 3\zeta(t) t_0^{-1} \int_0^{\rho_2} d\rho\, \rho^2 g^{(s)}(\tilde{t}). \qquad (2.12)$$

Invoking further Eqs. (1.20), (1.19), and (2.3), we represent the flux J_0 in the form

$$J_0 = (a/\pi)^{1/2} n_{1,\infty} t_0^{-1} (\zeta^{(s)}+1)^2 \exp(-F_{\rho_c{}^{(s)}}) 3\int^{\rho_2} d\rho\, \rho^2. \qquad (2.13)$$

Finally, we express the derivative $d\zeta(t)/dt$ in terms of the corresponding derivative of the critical radius $\rho_c(t)$ in accordance with (1.4), (1.5), and (2.3):

$$\frac{d\zeta(t)}{dt} = -\frac{2}{3} a[\zeta(t)+1]\rho_c^{-2}(t) \frac{d\rho_c(t)}{dt}. \qquad (2.14)$$

Using (2.10)–(2.14), we obtain from (2.1) the required equation for the critical radius:

$$\frac{d\rho_c(t)}{dt} = \frac{9}{2} (a\pi)^{-1/2} t_0^{-1} \rho_c{}^2(t) \frac{(\zeta^{(s)}+1)^2}{\zeta(t)+1} \exp(-F_{\rho_c{}^{(s)}}) \int_0^{\rho_c} d\rho\, \rho^2 \left\{ \frac{\zeta(t)[\zeta(\tilde{t})+1]^2}{\zeta(\tilde{t})[\zeta^{(s)}+1]^2} \exp(F_{\rho_c{}^{(s)}} - F_{\rho_c(\tilde{t})}) - 1 \right\}. \qquad (2.15)$$

3. Relaxation to Steady State

We consider evolution times of the system of drops for which the difference $\rho_c(t) - \rho_c^{(s)} \equiv \delta\rho_c(t)$ becomes small, so that in Eq. (2.15) we can go over to the approximation linear in $\delta\rho_c$. Ignoring also the terms small by virtue of the conditions (1.7) and (1.8) when we linearize with respect to $\delta\rho_c$, we obtain from (2.15) and (2.9)

$$\frac{d\delta\rho_c(t)}{dt} = -c\int_0^{\rho_2} d\rho\, \rho^2 [\delta\rho_c(t-t_\rho) + p\delta\rho_c(t)], \qquad (3.1)$$

where

$$c = (a/\pi)^{1/2} 3 t_0^{-1} (\zeta_{(s)}^{(s)}+1) (\rho_c^{(s)})^3 \exp(-F_{\rho_c{}^{(s)}}), \qquad (3.2)$$

$$p = (\zeta^{(s)}+1)/\zeta^{(s)}(\rho_c)^3 \qquad (3.3)$$

and $t_\rho = t_0\rho/\zeta^{(s)}$ is the time of growth of a drop to the size ρ for constant value $\zeta^{(s)}$ of the degree of supersaturation.

We investigate the spectrum of relaxation times of Eq. (3.1). The solution of (3.1) has the form of a superposition of the expressions

$$\delta\rho_c(t) = \mathrm{const}\, e^{-qt}, \qquad (3.4)$$

where q are unknown numbers, in general complex. The real part of q is the reciprocal time of relaxation τ^{-1} to the investigated steady state, and the imaginary part is the frequency ω of possible oscillations of the critical radius near $\rho_c^{(s)}$. Substitution of the solution (3.4) in Eq. (3.1) leads to a dispersion relation for q:

$$q - c \int_0^{\rho_2} d\rho\, \rho^2 (e^{qt_\rho} + p) = 0. \tag{3.5}$$

We introduce convenient auxiliary variables z and z', defining them by

$$z = qt_{\rho_2}, \quad z' = qt_\rho. \tag{3.6}$$

In the new variables, Eq. (3.5) becomes

$$\Phi(z) + p/3z = l, \tag{3.7}$$

where

$$\Phi(z) = z^{-4} \int_0^z dz'\, z'^2 e^{z'}, \tag{3.8}$$

$$l = \zeta^{(s)}/t_0 c \rho_2^4. \tag{3.9}$$

We shall assume that for all values of z and $\Phi(z)$

$$|3z\Phi(z)| \gg p, \tag{3.10}$$

whose validity we shall prove in what follows. In this case, from the expression (3.7) we have

$$\Phi(z) = l. \tag{3.11}$$

We represent Eq. (3.11) as a system of equations for the real and imaginary parts of the function:

$$\operatorname{Re} \Phi(z) = l, \tag{3.12}$$

$$\operatorname{Im} \Phi(z) = 0. \tag{3.13}$$

If our linearization of Eq. (2.15) is to be valid, we require $\operatorname{Re} z > 0$. By virtue of this condition and the obvious equation $\Phi^*(z) = \Phi(z^*)$, it is sufficient to investigate the solutions of the system of equations (3.12) and (3.13) in the region $0 \le \arg z < \pi/2$.

Each of Eqs. (3.12) and (3.13) defines a family of curves on the complex plane. The curves corresponding to Eq. (3.12) depend parametrically on l; among the family of curves described by Eq. (3.13) there is the positive half-axis $\arg z = 0$. The points of intersection of the two families give the required solutions of Eq. (3.11).

If Eq. (3.11) has several roots, then the main contribution to the superposition of solutions (3.4) will be made by the term with the smallest damping, corresponding to the root with the smallest real part. It is only such roots in which we shall be interested in what follows.

The general properties of the real roots of Eq. (3.11) can be established directly from the form of the function $\Phi(z)$. Being positive on the complete real axis, the function $\Phi(z)$ increases unboundedly as $z \to +0$ and $z \to \infty$. In the region $z > 0$, the graph of the function is concave. At the point $z = z_0 \simeq 1.26$, it reaches its minimal value $\Phi(z_0) = l_0 \simeq 0.7$ in the region $z > 0$. Therefore, for $l > l_0$ in the region $z > 0$ Eq. (3.11) has two real roots z_1 and z_2 satisfying $z_1 \le z_0 \le z_2$. With decreasing l, the roots z_1 and z_2 first approach each other, coming together at the point z_0 when $l = l_0$, and they then move apart on the complex plane with further decrease in l. In Fig. 1, as an example, we show sections of four (including the real axis) curves in the complex plane on which $\operatorname{Im} \Phi(z) = 0$. The motion of the roots z_1, z_2 that we have described takes place first along the real axis to the point $z = z_0$; thereafter, the roots move symmetrically upward and downward along the curve identified by α. Besides the roots z_1, z_2 there are also complex roots on the curves γ, δ, etc. However, in all cases when a steady state is attainable these roots have a larger real part than the corresponding roots on curve α.

Associating each point of the interval $[0, z_0]$ and each point of the arc of the curve α for $\operatorname{Re} z \ge 0$ with the value $\operatorname{Re} \Phi(z) = l$, we establish the correspondence between the values of α and the corresponding values of the smallest real parts $\operatorname{Re} z(l)$ of the roots of Eq. (3.11). The graph of $\operatorname{Re} z(l)$ is shown in Fig. 2. The region $l \ge l_0$ corresponds to the interval $[0, z_0]$ of the real axis, the interval $l_0 > l > l_{cr} \simeq 0.15$ to the arc of the curve α. The range of values of $\operatorname{Re} z(l)$ is the interval $0 \le \operatorname{Re} z(l) \le z_0 \simeq 1.26$.
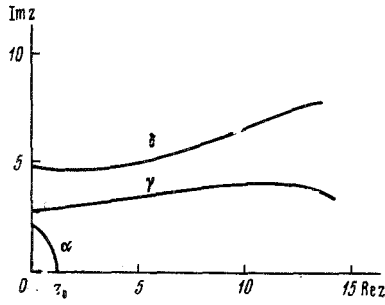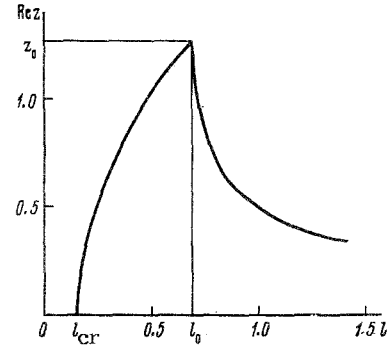
Fig.1

Fig.2

We now turn to the inequality (3.10). In accordance with (1.7), (1.8), and (3.3) the coefficient p is much less than unity in magnitude. For the regions of variation of $|z|$ and $|\Phi(z)|$ for complex z in Figs.1 and 2 we have $|3z\Phi(z)| \sim 1$. In the region of real values of z, using the lower bound $\Phi(z) \gtrsim (3z)^{-1}$, we obtain $|3z\Phi(z)| \gtrsim 1$. Thus, the relation (3.10) is indeed satisfied.

It follows from Fig.2 that the inequality

$$l > l_{cr} \simeq 0.15 \tag{3.14}$$

is the condition required if a steady state is to be attainable. We cast this condition in a more perspicuous form. To this end, we introduce a certain limiting radius ρ_m, defined by the relation $\zeta^{(s)} n_{1,\infty} = \int_0^{\rho_m} d\rho\, \rho^3 g^{(s)}$. It is obvious that the drops nucleated at the given degree of supersaturation $\zeta^{(s)}$ only slightly deplete the vapor until they reach radii of order ρ_m. By virtue of (2.10), ρ_m is given by $\rho_m^4 = 4(a/\pi)^{-1/2}(\zeta^{(s)})^2(\zeta^{(s)}+1)^{-2}\exp(F_{\rho_c}^{(s)})$. This last equation enables us to write the inequality (3.11) in the form $(\rho_2/\rho_m)^4 \ll 1$. Thus, when a steady state is attainable, the mass of the excess vapor appreciably exceeds the mass of the liquid phase in the system. In this sense, the conditions of formation of a steady condensation process are opposite to the conditions corresponding to transition to the coalescence stage [1].

In accordance with what we have said above and Eq. (3.6), the time of relaxation to the steady state is $\tau = t_{\rho_2}/\mathrm{Re}\, z(l)$. The change in the supersaturation in the system for values of the parameter l in the interval $l_{cr} < l < l_0$ can be characterized not only by the relaxation time but also by the oscillation period $\tau_\omega = 2\pi t_{\rho_2}/\mathrm{Im}\, z(l)$. The theory we have developed is valid if for all values of τ and τ_ω the condition of quasi-stationarity is satisfied, i.e., the relative change $\delta\rho_c(t)$ during the correlation time t_s is small. This condition can be written in the form of the two inequalities $t_s/\tau \ll 1$, $t_s/\tau_\omega \ll 1$. From (1.24), the expressions for τ, τ_ω, t_{ρ_2}, and from Fig.2, we find that these inequalities impose a lower bound on the possible values of ρ_2, namely,

$$\rho_2 \gg \rho_c^3 \zeta^{(s)}/(\zeta^{(s)}+1). \tag{3.15}$$

The condition (3.15) obviously agrees with the condition (1.15) previously imposed, but it is stronger.

Besides τ and t_{ρ_2}, we consider one further characteristic time τ_ζ, which we take equal to $\tau_\zeta = (\zeta^{(s)}+1)n_{1,\infty}/J_0$. This time is obviously required in the absence of condensation processes to raise the degree of supersaturation of the vapor in the system to the value $\zeta = \zeta^{(s)}$. Under conditions when a steady state is attainable, quite definite relations must be satisfied between the times t_{ρ_2} and τ_ζ and τ and τ_ζ. Indeed, it follows from (3.14), the definition of l, and Eq.(3.2) that $(\rho_c^{(s)})^3 \tau_\zeta/t_{\rho_2} > 0.15$. Hence, by virtue of (1.7) and (1.8), we have $\tau_\zeta/t_{\rho_2} \gg 1$. Thus, the supersaturation of the vapor should proceed much more slowly than the process of growth of the drops. In exactly the same way one can show that $\tau_\zeta \gg \tau$ for $l > l_{cr}$ and $\tau_\zeta \sim \tau$ for $l \sim l_{cr}$. Therefore, for $l > l_{cr}$ the total time of evolution of the system from the beginning of the supply of vapor to the establishment of the steady state is made up in the general case of the times τ_ζ and τ and in order of magnitude is τ_ζ.

Finally, we discuss the compatibility of the inequalities (3.14) and (3.15). Choosing as independent variables ρ_2 and $\rho_c^{(s)}$, we write both inequalities in the form of conditions on ρ_2:

$$(\rho_c^{(s)})^3[1-\exp(-{}^2/_3 a/\rho_c^{(s)})]\ll\rho_2<[(\pi/a)^{1/2}/0.45]^{1/4}[1-\exp(-{}^2/_3 a/\rho_c^{(s)})]^{1/4}\exp[{}^1/_{12}a(\rho_c^{(s)})^2-{}^1/_4 a\gamma\rho_c^{(s)}]/(\rho_c^{(s)})^{3/4}, \qquad (3.16)$$

where we have used the expression (1.2). The condition (3.16) must be augmented by the inequality $\rho_c^{(s)}\geqslant\rho_c^{(\min)}$, where $\rho_c^{(\min)}$ is the minimal drop radius for which the conditions of applicability (1.7) and (1.8) of the macroscopic theory are still satisfied. We see from (3.16) that the presence of the leading microscopic correction for the work of formation of the drop surface leads at large supersaturations to an important bound on the possible values of ρ_2 for which a steady state arises in the system. Basically, for macroscopic values of ρ_2 the inequality (3.16) remains compatible up to values of the critical radius satisfying $\rho_c\sim\rho_c^{(\min)}$. Thus, the considered steady process of homogeneous condensation can be established for practically all the values of the degree of supersaturation admissible in the macroscopic theory.

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DYNAMICS OF MODELS WITH TENSOR ORDER PARAMETER NEAR AN ISOLATED POINT ON THE PHASE-TRANSITION CURVE

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The dynamics of models with tensor order parameter near an isolated point on the phase-transition curve is investigated. The interaction between the order parameter and the densities of the conserved quantities is considered. The method using the dynamical renormalization group and ε expansion is used to calculate the lowest ε approximations for the dynamical critical exponent and the exponent of the anomalous behavior of the Onsager coefficient.

1. In the present paper, we investigate the dynamics of the Q model [1] and the Askin–Teller–Potts model (ATP model) [2, 3] near an isolated point on the phase-transition curve. In these models, the order parameter is a P-dimensional symmetric traceless tensor, diagonal in the ATP model. The special case of the Q model for P = 3 describes the transition of an isotropic liquid into a nematic liquid crystal. The ATP model is a generalization of the Ising model and describes a system in which each site on the lattice can be in one of P states. In addition, the results of the ATP model in the limit P → 1 describe the percolation problem [4]. The static behavior of the models near the singular critical point associated with the vanishing of the coefficient of the cubic interaction in the effective Hamiltonian was considered in [5, 6].

In the paper presented here, using a static scale-invariant solution, we investigate the dynamics of these models in $(4 - \varepsilon)$-dimensional space. We use a method that is a generalization of the renormalization-group and ε-expansion method [7, 8] to the field of dynamical phenomena [9, 10]. We find the quadratic terms in the ε expansion of the dynamical critical exponent and the exponent of the anomalous behavior of the Onsager coefficient for both models for all P. We also discuss the question of the influence of the interaction between the order parameter and the densities of the conserved quantities on the critical dynamical order parameter. We show that the interaction with the angular momentum density can be ignored, but that the interaction with the energy density in general influences the relaxation of the order parameter. For the latter case, in the first ε approximation, we obtain static and dynamic recursion relations, which we solve explicitly for the Q model with P = 3. For this case we show that the relaxation rates of the energy and the order parameter are equal.

2. We consider the possibility of extending the set of fundamental variables and take into account the interaction between the order parameter and the densities of the conserved quantities. The fluctuations of the particle number density can be ignored, since real liquid crystals behave like incompressible fluids.