On the definition of the disjoining pressure of a wedge-shaped film

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Erratum

On the definition of the disjoining pressure of a wedge-shaped film

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An erratum is corrected concerning the definition of the disjoining pressure of a wedge-shaped film in the previous publication of the authors.

In the text of our previous article [1], it was assumed that the structure of the pressure tensor corresponds to that of the metric tensor of a system in the absence of external fields. This assumption is intuitive and has never been proven, and turns out not always to be valid. In particular, it is invalid in the case of the wedge-shaped film considered in our article, among other examples.

Let a wedge-shaped film of phase $\alpha$ form between walls $\beta$ and $\gamma$ (figure 1). The cylindrical coordinate system $r$, $\varphi$, $z$ (with the $z$ axis along the edge) diagonalizes the metric tensor, but does not diagonalize the pressure tensor in this case. Indeed, in view of the fact that surface layer overlapping increases to the edge vertex, the vector $p$ of the force applied to the unit surface of wall $\beta$ is evidently not normal to the wall and should have a certain inclination to the left where the attraction from wall $\gamma$ is stronger than on the right. This means that not only $p_{\varphi\varphi}$, but also the component $p_{r\varphi}$ of the pressure tensor is not zero, and the known mechanical equilibrium condition holds

$$\frac{1}{r} \frac{\partial p_{\varphi\varphi}}{\partial \varphi} + \frac{2p_{\varphi\varphi}}{r} + \frac{\partial p_{r\varphi}}{\partial r} = 0 \quad (1)$$

throughout the wedge-shaped film.

It is of note that equation (1) itself does not prohibit the condition

$$\frac{\partial p_{\varphi\varphi}}{\partial \varphi} = 0, \quad (2)$$

which can be satisfied if the condition

$$\frac{2p_{\varphi\varphi}}{r} + \frac{\partial p_{r\varphi}}{\partial r} = 0 \quad (3)$$

is fulfilled separately. Moreover, one can show that equation (2) is inevitably fulfilled in the middle of the wedge. Let us denote the values of the normal component $p_{\varphi\varphi}$ at the wedge walls $\beta$ and $\gamma$ as $p_{\varphi\varphi}^\beta$ and $p_{\varphi\varphi}^\gamma$, respectively. If one alternatively rotates wall $\beta$ or wall $\gamma$ in the direction to the opposite wall by the same elementary angle $d\varphi$, the two final states are obviously physically indistinguishable from each other if the wall areas are equal. This means that elementary work done per unit area $p_{\varphi\varphi}^\beta r d\varphi$ and $p_{\varphi\varphi}^\gamma r d\varphi$ are equal, and, consequently, $p_{\varphi\varphi}^\beta = p_{\varphi\varphi}^\gamma$ irrespective of whether walls $\beta$ and $\gamma$ are of the same or different nature. If $p_{\varphi\varphi}$ is a variable quantity, symmetrical character of the above equality requires the existence of the condition expressed in equation (2) in the middle of the wedge.

If, however, $p_{\varphi\varphi}$ is a function of $\varphi$ (even a symmetrical one), the definition of the disjoining pressure should be referred to a certain specified value of the function.

Considering the transitional zone of a meniscus [1], we used the the normal pressure at a flat solid surface for the definition of the disjoining pressure. The same can

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Figure 1. A wedge-shaped film.
be done in the case of a wedge-shaped film. Since the wedge walls are flat, there is no contribution from capillary pressure at the wall, and the pressure variation can be assigned only to the disjoining pressure. So we define the disjoining pressure $\Pi$ of a wedge-shaped film as

$$\Pi \equiv p^\beta - p^\alpha = p^\gamma - p^\alpha$$  \hspace{1cm} (4)$$

where $p^\alpha$ is pressure in the bulk phase $\alpha$ at a broad part of the wedge-shaped film. Equation (4) rectifies the definition of the disjoining pressure of a wedge-shaped film given in [1].

**Reference**